

# Design of Double-Passed Arrayed Waveguide Gratings for Generation of Flat Topped Femtosecond Pulse trains

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## Abstract

We present a theoretical investigation of femtosecond flat-topped pulse sequence generation using an Arrayed Waveguide Grating in a double passed configuration. We present low insertion loss designs using phase-only filters designed using numerical optimization. The designs are also analyzed for robustness to fabrication errors.

## Index Terms

Arrayed Waveguide Grating, optical pulse generation, phase only filter, Wavelength division multiplexing.

## I. INTRODUCTION

The Arrayed-Waveguide Grating (AWG) is now a well-known technology which is deployed for applications in Wavelength Division Multiplexed (WDM) channel demultiplexers and optical routers [1]–[6]. A few recent experiments [7]–[10] have demonstrated a completely new functionality of the AWG. In the initial experiment by Leaird et al. [7], the AWG was read-out by a femtosecond pulse to yield a burst consisting of tens of femtosecond pulses under a Gaussian envelope. The pulse repetition rate is set by the delay increment in the waveguide

array section and can be in the several hundred GHz to THz range. A subsequent experiment introduced loss engineering (i.e., adding attenuation to appropriate waveguides in the waveguide array) to shape the pulse train envelope to have a flat-topped (equal intensity) character [9]. By combining two outputs of such a loss-engineered AWG read-out by a high-rate modelocked fiber laser, generation of a 500 GHz continuous pulse train was achieved [10]; this constituted a 42-fold repetition rate multiplication factor compared to the 11.9 GHz rate of the modelocked laser. Another experiment which recombined multiple outputs of the AWG by using double-pass geometry showed promise for a much higher repetition rate multiplication factors in the range of a several hundred, with the possibility of fundamentally low insertion loss [8]. However, loss engineering directly gives rise to excess insertion loss and is not able to provide for a flat-topped intensity envelope in the double-pass geometry. The principal contribution of this paper is the design of AWGs which when employed in a double-passed configuration yield long pulse trains with a square envelope, yet with much higher energy efficiency than that obtained by loss engineering. This is achieved via numerical optimization [13], using as parameters the phases of the individual waveguides that constitute the AWG. We will also demonstrate that the proposed approach allows a systematic tradeoff between the various design objectives such as the flatness of the pulse train temporal envelope, the length of the envelope, and energy loss. Numerical simulations establish that the designs are “robust,” i.e., the performance of the designs degrades gracefully in the presence of fabrication errors, thus encouraging their practical implementation. This paper is structured as follows. In section II we review relevant results from the previous work on the time-domain response of the AWGs while introducing a mathematical framework for the AWG from a systems perspective convenient for the optimization work. Section III defines our optimization approach. Section IV presents the simulation results for pulse sequences designed via our optimization approach. Simulation results testing the robustness of design waveforms are also discussed. We conclude with section V.

We note that others have investigated AWG designs in which individual waveguide phases are manipulated, e.g., [11], [12]. In the prior work the focus is on the frequency domain response of the AWG, e.g., tailoring the passband shapes or the dispersion. In contrast, in the current paper for the first time we investigate the design of waveguide phases such that a desired time-domain response is obtained, in particular the double-pass intensity profile.

## II. TIME-DOMAIN RESPONSE OF THE AWG AND MATHEMATICAL FRAMEWORK

We consider an AWG with light coupled in via a single input guide, with  $2M + 1$  waveguides in the waveguide array section. We make the following assumptions:

- 1) The delay increment  $\tau$  between two consecutive waveguides in the array is a constant.
- 2) We consider the response to an individual input pulse with pulse width  $t_p$  less than the delay increment per guide,  $t_p < \tau$ . This means that laser bandwidth exceeds the AWG's free spectral range (FSR), which is determined by the delay increment through the well known relationship  $FSR = \frac{1}{\tau}$ .
- 3) The dispersion in the individual waveguides in the waveguide array is negligible.

Let the input pulse be  $e_{in}(t)e^{j\omega_c t}$ , with spectrum (Fourier transform)  $E_{in}(j(\omega - \omega_c))$ . With  $t_p$  denoting the input pulse width, the bandwidth of  $E(j\omega)$  is proportional to  $1/t_p$ . Under the assumptions, at each output waveguide, the phase match condition is satisfied for a series of frequencies spaced by the FSR, but with a slight frequency shift (less than the FSR) from one output guide to the next. Thus, the spectrum of the signal at the  $n$ th output guide of the AWG is then given by

$$E_{out}^{(n)}(j\omega) = \sum_{k=-\infty}^{\infty} E_{in}(j(\omega - \omega_c))S(j(\omega - \omega_n - \frac{2\pi k}{\tau})), \quad (1)$$

where  $\omega_n$  denotes the center frequency (taken within a single diffraction order of the AWG) corresponding to the  $n^{th}$  output waveguide, and  $S(j(\omega - \omega_{pm}))$  represents the bandpass characteristics of the AWG around any frequency  $\omega_{pm}$  where phase match occurs. This is well known from numerous studies of the AWG as a frequency demultiplexing device. It is convenient to choose  $\omega_c = \omega_0$ , i.e.,  $\omega_c$  is chosen equal to a pass frequency for the central ( $n = 0$ ) output guide. For future reference, we let  $s(t)$  denote the inverse Fourier transform of  $S(j\omega)$ , and  $T$  denote the effective width (i.e., temporal extent) of  $s(t)$ . The bandwidths of  $E_{in}(j\omega)$  and  $S(j\omega)$  are denoted  $\Delta\nu$  and  $\Delta f$ , and are proportional to  $1/t_p$  and  $1/T$  respectively. A representative plot of  $E_{in}(j\omega)$  and  $S(j\omega)$ , under the assumptions, is shown in Fig. 1. Thus, at the  $n^{th}$  output waveguide, the AWG functions as a bank of narrow bandpass filters, centered at frequencies  $\omega_n + \frac{2\pi k}{\tau}$ , where  $k$  is an integer.

We next consider the temporal response at the  $n$ th output waveguide. From the inverse Fourier transform of (1)

$$e_{\text{out}}^{(n)}(t) = \sum_{k=-\infty}^{\infty} \left( e_{\text{in}}(t) e^{j\omega_0 t} \right) \star \left( s(t) e^{j\omega_n t} e^{j\frac{2\pi k}{\tau} t} \right), \quad (2)$$

where ‘ $\star$ ’ denotes convolution. Using the identity

$$\sum_{k=-\infty}^{\infty} e^{j\frac{2\pi k}{\tau} t} = \tau \sum_{k=-\infty}^{\infty} \delta(t - k\tau), \quad (3)$$

we have

$$\begin{aligned} e_{\text{out}}^{(n)}(t) &= \tau \left( e_{\text{in}}(t) e^{j\omega_0 t} \right) \star \left( \sum_{k=-\infty}^{\infty} s(t) e^{j\omega_n t} \delta(t - k\tau) \right) \\ &= \tau \left( e_{\text{in}}(t) e^{j\omega_0 t} \right) \star \left( \sum_{k=-\infty}^{\infty} s(k\tau) e^{jk\omega_n \tau} \delta(t - k\tau) \right) \end{aligned} \quad (4)$$

Note here that for our choice of  $\omega_c = \omega_0$ , coinciding with the transmission peak of the central output guide, we have  $\omega_0 \tau = 2m\pi$ , which is the condition for constructive interference. Thus, from a time domain perspective the AWG as viewed by the  $n^{\text{th}}$  output guide behaves, in effect, as a collection of delay lines, with a delay increment of  $\tau$ , a phase shift increment of  $(\omega_n - \omega_0)\tau$ , and a transmission amplitude of  $s(k\tau)$  for the  $k^{\text{th}}$  delay line. As sketched in Fig. 1, for a short pulse input, the output is a sequence of pulses with pulse spacing  $\tau$ . The amplitude of the  $k^{\text{th}}$  pulse,  $s(k\tau)$ , is equal physically to the amplitude of the field component coupled from the input guide through the  $k^{\text{th}}$  guide in the waveguide array to the  $n^{\text{th}}$  output guide. The  $e^{jk(\omega_n - \omega_0)\tau}$  term acts to pick out frequency components for the  $n^{\text{th}}$  output guide that are shifted by  $\omega_n - \omega_0$  relative to those in the central guide. Since  $s(k\tau)$  is independent of output guide number, pulse sequences with identical intensity profiles but with slight shifts in their frequency combs are produced at each of the output guides, as demonstrated in [7]. The different  $s(k\tau)$  can be modified by inserting various attenuations into the different guides in the array; this is the basis for waveform control via loss engineering, as demonstrated in [9].

Consider now a small change  $\Delta\tau_k$  in the delay of the  $k^{\text{th}}$  waveguide where  $\Delta\tau_k$  is taken to be on the order of one optical cycle or less. This would physically correspond to altering the length of the  $k^{\text{th}}$  waveguide by a small path length  $\delta L_k$  where  $\delta L_k$  is on the order of one wavelength

or less. This would imply that  $k\tau \rightarrow k\tau + \Delta\tau_k$ . Consequently the output will be:

$$\begin{aligned} e_{\text{out}}^{(n)}(t) &\approx \tau(e_{\text{in}}(t)e^{j\omega_0 t}) \star \left( \sum_{k=-\infty}^{\infty} s(k\tau) e^{j\omega_0(k\tau + \Delta\tau_k)} e^{j(\omega_n - \omega_0)(k\tau + \Delta\tau_k)} \delta(t - k\tau) \right) \\ &\approx \tau(e_{\text{in}}(t)e^{j\omega_0 t}) \star \left( \sum_{k=-\infty}^{\infty} s(k\tau) e^{j\omega_0 \Delta\tau_k} e^{jk(\omega_n - \omega_0)\tau} \delta(t - k\tau) \right) \end{aligned} \quad (5)$$

where we have used  $e^{j\omega_0 k\tau} = 1$  and made the approximation  $|(\omega_n - \omega_0)\Delta\tau_k| \ll 1$ , which is valid as long as  $|\omega_n - \omega_0| \ll \omega_0$ . Thus, in summary, the impulse response of the envelope from the input to the  $n^{\text{th}}$  output guide can be modeled as

$$h^{(n)}(t) = \tau \sum_{k=-M}^M b_k e^{jk(\omega_n - \omega_0)\tau} \delta(t - k\tau) = \tau \sum_{k=-M}^M b_k e^{j(\omega_n - \omega_0)t} \delta(t - k\tau), \quad (6)$$

where  $b_k = s(k\tau)e^{j\omega_0 \Delta\tau_k}$  and  $b_k = 0$  for  $|k| > M$  (since there are only  $2M + 1$  waveguides, by assumption). The important point being small changes in the delay in the  $k^{\text{th}}$  waveguide affording the possibility of freely changing the *phase* of  $b_k$ .

In the double-passed configuration, these pulse bursts observed at different output guides are recombined into a single optical channel by returning them through the AWG using a mirror and a suitable set of delays. For simplicity, we restrict our attention to the case of an AWG where the input and output slab regions are identical. Then, from symmetry, the contribution of the double-pass envelope response corresponding to the  $n^{\text{th}}$  output waveguide is simply the auto-convolution  $g^{(n)} = h^{(n)} \star h^{(n)}$ . Thus, the contribution of the  $n^{\text{th}}$  output waveguide to the double-passed response is

$$g^{(n)}(t) = \tau^2 e^{j(\omega_n - \omega_0)t} \sum_i a_i \delta(t - i\tau), \quad (7)$$

where

$$a_i = \sum_k b_k b_{i-k} \quad (8)$$

Such autoconvolution behavior has been observed experimentally- compare measured intensity profiles in Fig. 2 for a loss-engineered device for single and double passed operation, respectively (reprinted from [8]). Clearly the flat-topped intensity profile is lost upon double-passing the device. Our design goal in this paper will be to achieve flat-topped behavior in double-pass by appropriately designing the  $b_k$  in (6). In the experiment of [8], pulse trains from various output guides were recombined with delays such that the contributions from different output guides are not overlapped in time. Under this condition the different pulse trains have identical intensity

profiles and add incoherently. From the design perspective, it is then sufficient to consider the double-pass waveform contributed by a single output guide. This is the approach we will adopt in the remainder of this paper.

### III. OPTIMIZATION APPROACH

We now explore the problem of designing AWGs where a single input pulse, after a double-pass, yields a train of pulses whose heights are approximately the same, i.e., “flat-topped.” We will restrict our attention to the case when the output from a single output waveguide is returned through the AWG using a mirror. Our design objectives are the following:

- The double-passed output waveform should be close to an ideal square train of pulses, i.e., with a constant amplitude in the temporal region of the pulse train, and zero amplitude elsewhere.
- The energy efficiency should be high, i.e., the energy of the double-passed output must be as close as possible to the energy of the input pulse.

We now proceed towards formally translating these design objectives into a numerical optimization problem. Let  $K$  denote the number of desired pulses in the double-passed output. For convenience, we assume that  $K$  is odd, and let  $K = 2l + 1$ . Without loss of generality, we assume that the output pulse train amplitude is normalized to unity.

#### *Constraints*

- 1) The first design objective, that the output pulse train be as close to an ideal square pulse train, yields the constraint:

$$\begin{aligned} \left| |a_i| - 1 \right| &\leq \epsilon_1, & -l \leq i \leq l \\ |a_i| &\leq \epsilon_2, & \text{otherwise,} \end{aligned} \tag{9}$$

where the variables  $a_i$  characterize the double-passed output via (8). The variables  $\epsilon_1$  and  $\epsilon_2$  represent the deviation from ideality of the output pulse train in the temporal region of the pulse train and outside (henceforth referred to as the “wings”), respectively. A smaller value of  $\epsilon_1$  ensures a flatter pulse train, while a smaller value of  $\epsilon_2$  ensures that the pulse train cuts off sharply.

- 2) The second design objective, that the transmission losses be low, is handled indirectly in the spectral domain. Rather than directly address the insertion loss, we connect the insertion

loss with the shape of the double-passed transfer function magnitude, which exhibits a strong peak at frequencies  $\omega_n + \frac{2\pi k}{\tau}$ . Intuition suggests that the excess insertion loss may be minimized by peaking the spectral energy around these frequencies. A formulation for computing the excess insertion loss will be discussed later. However, for a standard AWG, it is well known that the wavelengths demultiplexed in one AWG can be recombined by passing through an identical second AWG (or double-passing back through the first AWG). Therefore, in order to minimize excess loss in the modified AWGs we wish to design, one should seek to retain the strongly peaked spectral character of the standard AWG. We implement this idea in our optimization problem by defining a small band called the “in-band” around the peak frequencies. We take  $\alpha$  as the ratio of the energies contained outside and inside the in-band. Then, the spectral peaking constraint, which is an indirect enforcement of the insertion loss constraint, can be formulated as:

$$\int_{\text{out-band}} |H(j\omega)|^2 d\omega \leq \alpha \int_{\text{in-band}} |H(j\omega)|^2 d\omega, \quad (10)$$

where  $H(j\omega)$  is the Fourier transform of the impulse response function of the central output waveguide, and represents its output spectral properties. The width of the in-band is chosen to be the 10 dB width of the Gaussian transmission of the unperturbed AWG. As  $\alpha$  takes on smaller values, there is sharper peaking of energy inside the in-band, leading to a decrease in the excess insertion loss.

### *Optimization variables*

Recall that our design freedom lies in the ability to change the waveguide lengths, and consequently the *phase* of the complex numbers  $b_k$  that characterize the single-pass transfer function (see (6)). We could add a degree of freedom in our design by letting the amplitude  $|b_k|$  also vary. However that would involve further addition of insertion loss to each waveguide, thereby conflicting with our energy efficiency requirement. *In fact, it will be pointed out that the our phase profile based optimization leads to a better performance (more than 3dB) in terms of excess insertion loss with respect to loss engineering reported in [9].* Therefore in this paper we will adjust only the phases of the  $b_k$ . The *magnitudes* of  $b_k$  are constrained, from the physics of propagation through the Free Propagation Region (FPR), to be approximately Gaussian, with

the central waveguide collecting the maximum energy. Thus, we have the constraint

$$|b_k| = b_0 e^{-\frac{k^2}{\gamma^2}}, \quad k = -M, -(M-1), \dots, M-1, M, \quad (11)$$

where  $\gamma$  is a parameter that controls the width of the Gaussian profile. Writing  $b_k = |b_k|e^{j\phi_k}$ , the optimization variables are the phase terms  $\phi_k$ .

### *The optimization problem*

Recall that  $\epsilon_1$  and  $\epsilon_2$  control the deviation of the output pulse train from the ideal square train. We take  $\epsilon_1 = w_{\text{FT}}\epsilon$ , and  $\epsilon_2 = w_{\text{WI}}\epsilon$ , so as to have a single variable  $\epsilon$ ; the parameters  $w_{\text{FT}}$  and  $w_{\text{WI}}$  reflect the relative weights that we assign between the desired properties of pulse train *flatness* and sharp cut-off of the pulse train *wings*. Then, we fix the values of:

- $K$ , the desired number of pulses in the double-passed output;
- the parameter  $\alpha$  that controls the ratio of the out-of-band to in-band energy (see (10));
- the parameter  $\gamma$  that characterizes the shape of the Gaussian profile of the magnitude of  $b_k$  (see (11));
- the parameters  $w_{\text{FT}}$  and  $w_{\text{WI}}$  that determine  $\epsilon_1$  and  $\epsilon_2$  in (9), thus characterizing the trade-off between pulse train flatness and cut-off,

and solve the optimization problem

$$\begin{aligned} &\text{minimize: } \epsilon \\ &\text{subject to: } (9), (10), \text{ and } (11). \end{aligned} \quad (12)$$

*Given a set of design parameters  $K, \alpha$ , and  $\gamma$ , one calculates the phase profile following optimization which yields the optimized phase profile as shown in the block diagram in Fig. 3.*

### *Design evaluation*

By numerically solving a series of optimization problems (12) for various values of  $K$ ,  $\alpha$ ,  $\gamma$ ,  $w_{\text{FT}}$  and  $w_{\text{WI}}$ , we may systematically explore the interplay between the various parameters that affect the constraints. This in turn enables us to study the tradeoff between the conflicting objectives of squareness of the output pulse train and low insertion loss.

The following quantitative measures will be used to evaluate the designs obtained:

- How close the output pulse train is to the ideal square train is captured by the quantities  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$ , defined as

$$\delta_{\text{FT}} = \max_{l \leq i \leq l} | |a_i|^2 - 1 |, \quad \delta_{\text{WI}} = \max_{l > |i|} |a_i|^2. \quad (13)$$

$\delta_{\text{FT}}$  serves to quantify how flat-topped the output pulse train is, and  $\delta_{\text{WI}}$  measures the the electric field intensity in the wings, i.e., it quantifies how sharply the pulse train cuts off.

- The insertion loss is computed as follows. The single-pass impulse response  $h^{(n)}$  is given by (6), with transfer function

$$H^{(n)}(j\omega) = \tau \sum_{k=-M}^M b_k e^{jk(\omega_n - \omega_0)\tau} e^{-jk\omega\tau}. \quad (14)$$

Specializing to the central output guide ( $\omega_n = \omega_0$ ), which is sufficient for our purposes, we have

$$H^{(0)}(j\omega) = \tau \sum_{k=-M}^M b_k e^{-jk\omega\tau}. \quad (15)$$

We will henceforth simplify our notation by writing  $H(j\omega)$  instead of  $H^{(0)}(j\omega)$ ; the implication is that we are specializing to the central output guide, although as discussed earlier all of the output guides give very similar responses. The double-pass impulse response is given by the convolution  $g^{(n)} = h^{(n)} \star h^{(n)}$ , and consequently the double-pass transfer function is  $G(j\omega) = H^2(j\omega)$ . Note that  $H(j\omega)$  and  $G(j\omega)$  are periodic in  $\omega$ . With the assumption that the input spectrum is broad (i.e, the input pulse can be well approximated by an impulse function), the power from one output waveguide double-passed through the AWG is simply proportional to

$$P_d = \int_{-\frac{\pi}{\tau}}^{\frac{\pi}{\tau}} |H(j\omega)|^4 d\omega \quad (16)$$

Here we have calculated only the power in one period of  $H(j\omega)$ ; for a broad flat input spectrum, the total power is just the power per period times the number of periods excited by the input pulse. Also, as explained earlier, we assume that if multiple outputs are double-passed through the AWG, then their relative delays are sufficient that there can be no temporal overlap. This ensures that the power contributions from each of them can be added, so that (16) gives the double-passed power transmission per frequency per output guide. Consequently we can quantify the excess insertion loss with respect to the standard

AWG device in dB as:

$$I_{\text{loss}} = 10 \left( \log \int_{-\frac{\pi}{\tau}}^{\frac{\pi}{\tau}} |H(j\omega)|^4 d\omega - \log \int_{-\frac{\pi}{\tau}}^{\frac{\pi}{\tau}} |H_0(j\omega)|^4 d\omega \right), \quad (17)$$

where  $H_0(j\omega)$  is the output pass-band spectrum of the untruncated AWG device (infinite number of guides in the array section) with the same excitation profile but with no added phases.

#### IV. SIMULATION RESULTS

We solve a series of optimization problems (12), using the standard local nonlinear optimization algorithm implemented in the MATLAB optimization toolbox [14], [15], with the following choices of parameters:

- *Earlier experiments [7]–[9] dealt with the generation of 15 to 25 pulses per output waveguide. We thus consider two representative values for the number of output pulses:  $K = 11$  and  $K = 19$ . (By trial and error, the corresponding number of waveguides in the AWG were chosen to be  $2M + 1 = 19$  and  $2M + 1 = 39$  respectively. These choices are also motivated by experiments [7], [9] that typically employed 20 or more waveguides. )*
- *We consider three possible choices for the parameter  $\gamma$  that controls the Gaussian profile of the magnitude of  $b_k$  (i.e., the excitation profile):  $\gamma = 0.7M, 0.83M, M$ .  $\gamma$  is a typical device parameter, that governs focusing properties. In our case a higher value of  $\gamma$  corresponds to truncation/blocking of waveguides near the trailing edge of the FSR mode incident on the array.*
- *We consider four values for the parameter  $\alpha$  that controls the ratio of the out-of-band to in-band energy (see (10)):  $\alpha = 0.15, 0.25, 0.35, 0.45$ . The use of  $\alpha$  is proposed in order to formulate the excess insertion loss constraint. Choice of the above four values is followed by several trials, and depicts consistent trade-off between spectral and temporal properties, while increasing  $\alpha$  beyond the value of 0.45 neither illustrates any trend better nor yields better results.*
- *We consider a range of values for the parameters  $w_{\text{FT}}$  and  $w_{\text{WI}}$  that reflect the relative weights that we assign between pulse train flatness and pulse train cut-off. Justification of parameters such as these or estimates of  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$  are judged using an eyeball measure of the temporal profiles obtained.*

For every parameter combination, the solution to the optimization problem (12) yields a locally optimal design, that is, a set of values for the phases  $\phi_k$  (or in turn, a corresponding set of values of waveguide path length variations) that minimizes a measure of the deviation of the output pulse train from the ideal square pulse train. Owing to the “local” nature of the optimization, different runs of the optimization program with different choices of initialization of the optimization variables would yield different “optimal” solutions. To address this issue, we run the same optimization program with 50 randomly chosen initial conditions and pick the best solution, which serves as an approximation to the “globally” optimal solution.

It is impossible to present the results obtained by simultaneously varying all the parameters that characterize problem (12). We instead present a series of plots, henceforth referred to as *tradeoff curves*, that present the performance of the optimal designs obtained by holding a subset of the parameters at a constant value and varying the remaining parameters [16]. More specifically, we hold  $\gamma$  and  $\alpha$  constant, and by varying the weight parameters  $w_{\text{FT}}$  and  $w_{\text{WI}}$  systematically, obtain a series of optimal designs by numerically solving (12). The values of  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$  are then plotted for the optimal designs, capturing the tradeoff between the conflicting design requirements of pulse train flatness and pulse train cut-off.

From the definition of the  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$  in (13) it is desirable to have both  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$  small. Thus, given a choice of two values for a parameter, say  $\gamma$ , the one for which the tradeoff curve is closer to the origin represents a better choice. Using this guideline, we pick one choice for both of the other parameters  $\alpha$  and  $\gamma$ , and from an examination of the resulting tradeoff curve, choose one optimal design that yields reasonable values of  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$ . This particular design is further analyzed for its time-domain performance. As we have noted a similar trend in the tradeoff curves for both the 19-waveguide/11-pulse and 39-waveguide/19-pulse problems, we present tradeoff curves for only the former problem. We however present the time-domain performance, designs (phases) and insertion loss trend for two best designs from each problem. The selection process for the best designs of the latter problem follows exactly the same trend (with the same values of the variable parameters  $\alpha$  and  $\gamma$ ) as the former problem.

An important practical issue is that of *robustness*: How gracefully does the performance of the optimal design degrade in the presence of variations in the optimization variables, i.e., the phase  $\phi_k$  of  $b_k$ ? We address this issue by allowing for random variation in  $\phi_k$  around the optimal value, and analyze the performance of the resulting designs.

### A. A sample tradeoff curve

We begin by presenting a typical tradeoff curve, i.e., a plot of  $\delta_{\text{WI}}$  versus  $\delta_{\text{FT}}$  for a fixed choice of parameters  $\alpha$  and  $\gamma$ , in Fig. 4 for the 11-pulse/19-waveguide system. Fig. 5 shows the output pulse trains corresponding to the selected points marked on the tradeoff curve in Fig. 4. Points along the trade-off curve are obtained by systematically varying the weights  $w_{\text{FT}}$  and  $w_{\text{WI}}$  as discussed earlier. We have set  $w_{\text{WI}} = 1$  and varied  $w_{\text{FT}}$  from 0.7 (point (d) on the trade-off curve) to 0.01 (point (a) on the trade-off curve). We have used the same set of weights for all the trade-off curves that are discussed in this paper. Notice the relative variation of pulse flatness  $\delta_{\text{FT}}$  between design (d) and design (a), corresponding to the extreme points of the curve. It can be seen that the pulse train cutoff  $\delta_{\text{WI}}$  varies only by a small margin (of 2%), with  $\delta_{\text{FT}}$  varying between 0.5% to 17% between the two points. Thus, a small sacrifice in  $\delta_{\text{WI}}$  can be traded off to realize a large reduction in  $\delta_{\text{FT}}$ .

### B. Effect of $\gamma$

Recall that the parameter  $\gamma$  characterizes the shape of the Gaussian profile of the magnitude of  $b_k$  (see (11)). We fix  $\alpha = 0.45$  (more on this shortly), and consider three values of  $\gamma$ :  $\gamma = 0.7M$ ,  $\gamma = 0.83M$ , and  $\gamma = M$ . Recall that we have chosen an odd number of waveguides numbered from  $-M$  to  $M$ . Figure 7 illustrates the tradeoff curve between  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$  for the 11-pulse 19-waveguide example ( $M = 9$ ). It is evident that the choice of  $\gamma = 0.83M$  yields the best performance (i.e., the tradeoff curve is closest to the origin). Fig. 6 shows the magnitude profile of the coefficients  $b_k$  for the three values of  $\gamma$ . The double-passed output envelope is directly affected by the  $b_k$  (via an autoconvolution); this explains why  $\gamma$  has a significant impact on  $\delta_{\text{FT}}$  and  $\delta_{\text{WI}}$ . Note that our definition of  $\gamma$  is with respect to (11) as a fraction of the number of waveguides  $M$  on each side with respect to the central waveguide. This would imply that as  $\gamma$  increases we observe an increase in the amount of truncation in the Gaussian profile as seen in Fig. 6. In our formalism for the excess insertion loss calculation Eqn. (17) we have defined  $H_0(j\omega)$  as the transmission of the untruncated AWG with the same width parameter  $\gamma$  and with no phase perturbation. This would imply that our formalism accounts for the insertion loss due to two factors namely: 1) due to truncation of the Gaussian excitation profile, 2) due to the degradation of the lineshape arising from the phase perturbations that serve as degrees of freedom for our design problem.

### C. Effect of $\alpha$

Recall that the parameter  $\alpha$  indirectly enforces the low insertion loss requirement via the spectral peaking constraint (10). Fixing  $\gamma = 0.83M$  (this is the best choice among the three values considered in Fig. 7), we choose four values for  $\alpha$ : 0.15, 0.25, 0.35, and 0.45, and plot the corresponding tradeoff curves in Fig. 8(a) and (b). A smaller value of  $\alpha$  enforces tighter spectral peaking (and a lower insertion loss) at the cost of larger deviation of the double-passed output pulse train envelope from the idea square train. Figs. 9 and 10 show respectively the output pulse train envelope and the normalized frequency spectrum of one optimal design for each value of  $\alpha$ . It is evident that  $\alpha = 0.15$  yields the most tightly-peaked frequency spectrum, however with the time-domain pulse envelope deviating the most from the ideal square (note that the pulse train cutoff  $\delta_{WI}$  for  $\alpha = 0.45$  in Fig. 8(a) is an order of magnitude smaller than that of  $\alpha = 0.15$  in Fig. 8(b)). This would imply that an optimal design, based on our analysis would be one representing point (A) on trade-off curve with  $\alpha = 0.45, \gamma = 0.83M$ . Designs corresponding to a particular point  $(\delta_{WI}, \delta_{FT})$  are not unique. There can be different designs (phases) with the same temporal performance (i.e, the same point on the trade-off curve). As a result of this, the calculated excess insertion loss varies from design to design. We have generated 10 designs that correspond to the points (A) (which have the flattest tops) on Fig. 8(a) and (b), and in each case noted the trend of excess insertion loss variation. We generate each design by iteratively generating the same point (point (A) in our case) on the trade-off curve. Owing to the local nature of the optimization, we obtain an entirely different design during each iteration. Table I depicts the trend of the calculated excess insertion loss (computed via (10)) for the various values of  $\alpha$  for the 11-pulse/19-waveguide system. It is seen from Table I that our optimal designs (point (A) on  $\alpha = 0.45, \gamma = 0.83M$  trade-off curve), for the 11-pulse/19-waveguide system have an excess insertion loss in the range of 4.83dB to 5.12dB. Note that as  $\alpha$  decreases, we perform significantly better spectrally (lower excess insertion loss from Table I), and at the same time deviating more significantly from our target waveform in time domain. *It should be pointed out that the above values for excess insertion loss is close to 3dB lesser than that of loss engineered AWG's [9], given the same number of (19) waveguides in the array.*

We pointed out in the beginning of this section that we shall display the selection process of the optimum design only for the 11-pulse/19-waveguide system, and choose the design with

TABLE I

TABLE OF RANGE OF CALCULATED EXCESS INSERTION LOSS FOR DESIGNS WITH THE FLATTEST TOP (POINTS MARKED (A) ON ALL TRADE-OFF CURVES SHOWN IN FIG. 8(A) AND (B)) FOR VARIOUS VALUES OF  $\alpha$  FOR THE 11-PULSE/19-WAVEGUIDE SYSTEM

$\alpha$	Excess Insertion Loss (dB)
0.15	0.87 to 1.05 dB
0.25	1.97 to 2.21 dB
0.35	3.28 to 3.57 dB
0.45	4.83 to 5.12 dB

similar parameters ( $\gamma = 0.83M$  and  $\alpha = 0.45$ ) for the 19-pulse, 39-waveguide system. A *similar set of design parameters yields optimum designs for the 19-pulse/39-waveguide system.*) Fig. 11 shows tradeoff curves for both 11-pulse/19-waveguide and 19-pulse/39-waveguide systems (in both cases with an optimum choice of the parameters  $\gamma = 0.83M$  and  $\alpha = 0.45$ ).

The point (A) marked on both curves represents the best design in terms of best temporal shape just by visual perception. We display these designs (phase profiles) in Tables II and III with the corresponding pulse trains (design 1 from Tables II and III) in Fig. 12. Notice (in case of the 11-pulse/19-waveguide system) that the pulse trains are nearly flat topped with pulse train flatness with less than 0.2% variation and a marginal intensity in the wing region (less than 4%). The 19-pulse/39-waveguide system seems to have a better performance with pulse train flatness having less than 0.2% variation and the wing region having a very marginal maximum intensity of 1.6%. We estimated the excess insertion loss for this system using our formulation (once again for designs represented by the points along the  $\gamma = 0.83M$  and  $\alpha = 0.45$  trade-off curve) to be in the range of 6.9 dB to 7.3 dB. Notice, that the insertion loss for the latter system is of the order of 2 dB more than the former. Also note that the pulse train cutoff in both cases is very sharp. We have thus obtained satisfactory designs after evaluating the interplay between several conflicting factors with the aid of tradeoff analysis. The optimal set of phases for both systems is shown in Tables II and III. It should be re-iterated that we are presenting two sample sets of optimal designs. In practice there could be several designs that yield the same temporal performance as in Fig. 12. We also summarize the performance of the optimum designs in Table IV.

TABLE II

TABLE OF PHASE V/S WAVEGUIDE NUMBER FOR TWO OPTIMAL DESIGNS FOR 11-PULSE/19-WAVEGUIDE SYSTEM

Waveguide number	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Phase (rad) (Design 1)	-1.3927	-0.3811	0.3378	1.5519	1.9063	1.8162	-0.0104	1.3155	0.0794	-0.6472
Phase (rad) (Design 2)	1.3587	0.8710	2.6465	0.3202	-0.7121	-1.3126	-0.3121	0.6825	1.7313	1.6703
Waveguide number	1	2	3	4	5	6	7	8	9	
Phase (rad) (Design 1)	-0.7715	-0.3234	-0.0312	-0.1849	-0.8111	-1.9700	2.5555	0.0999	1.9277	
Phase (rad) (Design 2)	0.9676	2.0144	1.5009	2.0087	0.3432	0.3622	-2.5299	-1.9733	-0.7468	

TABLE III

TABLE OF PHASE V/S WAVEGUIDE NUMBER FOR TWO OPTIMAL DESIGNS FOR 19-PULSE/39-WAVEGUIDE SYSTEM

Waveguide number	-19	-18	-17	-16	-15	-14	-13	-12	-11	-10
Phase (rad) (Design 1)	1.2374	2.6319	1.6258	1.6838	0.3552	0.0732	-0.8981	-0.5698	-0.6246	2.6777
Phase (rad) (Design 2)	0.1743	1.9003	1.8631	1.4263	-0.6353	-0.5443	0.5465	-1.1562	-0.4325	2.8728
Waveguide number	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Phase (rad) (Design 1)	-2.8338	-1.2577	-0.4392	-1.7758	0.0402	-0.9946	0.1441	-0.0576	-0.0434	0.9997
Phase (rad) (Design 2)	-2.9179	-0.9400	-1.1930	0.2965	-0.9942	0.1317	-0.3099	0.5422	0.4349	1.2946
Waveguide number	1	2	3	4	5	6	7	8	9	10
Phase (rad) (Design 1)	1.7070	1.0545	0.7736	0.6322	0.7855	0.5880	0.1880	-0.5094	-2.1038	2.8506
Phase (rad) (Design 2)	2.4359	1.4315	1.3115	1.7384	1.0463	1.3376	0.8171	0.6059	3.0852	-2.5429
Waveguide number	11	12	13	14	15	16	17	18	19	
Phase (rad) (Design 1)	2.3541	-1.0519	-3.0558	0.2634	-1.8500	1.0681	-1.3479	0.1871	1.0214	
Phase (rad) (Design 2)	2.9206	-1.7799	-0.9385	-0.0872	1.4768	-1.1989	2.8387	1.5080	1.9950	

TABLE IV

SUMMARY OF PERFORMANCE OF OPTIMAL DESIGNS (DESIGNS 1 AND 2 FROM TABLES II AND III) FOR

11-PULSE/19-WAVEGUIDE AND 19-PULSE/39-WAVEGUIDE SYSTEMS

pulse/waveguide system	11-pulse/19-waveguide	19-pulse/39 waveguide
$\delta_{FT}$	0.2%	0.1%
$\delta_{WI}$	4.6%	1.6%
Excess insertion loss (dB), Design 1	4.8 dB	7.26 dB
Excess insertion loss (dB), Design 2	5.08 dB	6.9 dB

#### D. Robustness

In practice, any implementation of the optimal design, i.e., waveguide length alterations, will not be exact. Thus, there remains the issue of robustness of the optimal design. We perform an empirical analysis of the optimal 11-pulse, 19-waveguide design by allowing random phase errors that are uniformly distributed over a realistic range of values. *Present fabrication techniques [17], [18] typically permit fabrication with path length jitters around a twenty-fifth of a wavelength. Theoretical attempts [19], [20] have focussed on frequency response, with major focus on minimizing crosstalk due to neighboring channels. However, our robustness analysis mainly deals with the degradation of temporal properties of the obtained flat topped pulse train with respect to random phase errors, while channel crosstalk is hardly the issue.* In our analysis, we perturb our designs with five ranges of path length jitter:  $\pm\frac{\lambda}{100}$ ,  $\pm\frac{\lambda}{50}$ ,  $\pm\frac{\lambda}{25}$ ,  $\pm\frac{\lambda}{16}$ ,  $\pm\frac{\lambda}{8}$ . For each range, we use a uniformly distributed random variable that is added to the design phases. The resulting degradation in performance is quantified by the pulse flatness measure  $\delta_{FT}$ . This is because  $\delta_{WI}$  changes from 4.5% to 8.5% which is very slight considering the initial value of  $\delta_{WI} = 0.045$ . For each allowable path length jitter range, we determine the *worst-case* degradation, by running 25 trials with randomly generated path length jitters, and picking the worst (in terms of  $\delta_{FT}$ ). As expected, the pulse flatness parameter  $\delta_{FT}$  increases with the path length jitter range, and this variation is quite noticeable. Table V summarizes the results in comparison with the unperturbed pulse train.  $\delta_{FT}$  changes from 0.5% to about 15% for a path-length jitter range that is as high as  $\pm\frac{\lambda}{16}$ ; and  $\delta_{FT} = 4\%$  in the  $\pm\frac{\lambda}{25}$  range. Fig. 13 illustrates the double-pass pulse train envelope for various path-length jitter ranges. It turns out that the excess insertion loss performance degrades marginally from 4.8dB in the unperturbed case to 4.84dB in the worst case of  $\pm\frac{\lambda}{8}$  jitter length jitters as well; this is evident from Fig. 14, which shows the unperturbed spectrum superposed with the spectrum corresponding to the worst-case degraded system due to a  $\pm\frac{\lambda}{8}$  jitter. Note that with path length jitters, the spectral shape changes only marginally, with the peaking maintained.

#### V. CONCLUSION

We have demonstrated that by appropriately modifying the phases of the waveguides of an AWG read-out by femtosecond pulses and used in a double-passed configuration, we may generate flat-topped pulse trains with much enhanced energy efficiency as opposed to conventional loss engineering. Moreover, the optimization framework employed offers the potential

TABLE V

TABLE OF PULSE TRAIN FLATNESS TO PULSE TRAIN CUTOFF FOR DIFFERENT EXTENTS OF PATHLENGTH JITTERS (WORST CASE).

$\delta L$	Percentage Deviation $\delta_{FT}$	Percent deviation $\delta_{WI}$
Unperturbed	0.5%	4.6%
$\pm \frac{\lambda}{100}$	0.5%	4.6%
$\pm \frac{\lambda}{50}$	2%	6%
$\pm \frac{\lambda}{25}$	4%	6%
$\pm \frac{\lambda}{16}$	15%	5.5%
$\pm \frac{\lambda}{8}$	28%	8.5%

to systematically study the tradeoff between the conflicting design requirements of pulse train flatness and pulse train cutoff. The designs obtained appear to be robust to fabrication tolerances corresponding to of phase jitter in the order of  $\pm \frac{\lambda}{16}$ , suggesting that the designs may be suitable for experimental implementation. As an alternative to double-passed operation, it should also be possible to cascade a pair of phase-engineered AWGs to obtain flat-topped pulse trains. Since then the two AWGs need not have identical designs, this would essentially double the number of optimization variables, which may lead to even better designs than the ones reported here.

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## Figure captions

Figure 1: Expected output spectrum and temporal profile for an arbitrary output guide of the AWG (modified from [7]). Note that an output pulse sequence is generated from every laser input pulse; only the response to a single laser pulse is depicted in the figure.

Figure 2: Output pulse trains. A) Single pass through the AWG. B) Double pass. C) Calculated output pulse train obtained by auto-convolving the single pass temporal response. (from Leaird et.al [8]).

Figure ??: Block diagram of the optimization procedure. Using the amplitude profile  $b_k$  and the optimized phase profile  $\phi_k$ , output target waveform, and spectral properties are evaluated via (8) and (15).

Figure 4: A sample trade-off curve. Here, we have set  $\gamma = 0.83M$  and  $\alpha = 0.45$ . We obtain the various points by setting  $w_{WI} = 1$  and varying  $w_{FT}$  gradually. Points marked (a) through (d) correspond to  $w_{FT} = 0.01, 0.04, 0.4, \text{ and } 0.7$  respectively.

Figure 5: Output temporal intensity profiles corresponding to points (a), (b), (c) and (d) on the trade-off curve. Figures (e), (f), (g) and (h) represent a closer look at the wing intensity corresponding to (a), (b), (c) and (d) respectively

Figure 6: Excitation Profile  $|b_k|$  for (a)  $\gamma = 0.7M$  (b)  $\gamma = 0.83M$  (c)  $\gamma = M$  for the 19-waveguide system.

Figure 7: Trade-off curve for (a)  $\gamma = 0.7M$  (b)  $\gamma = 0.83M$  (c)  $\gamma = M$  for the 11-pulse/19-waveguide system with  $\alpha = 0.45$ .

Figure 8: Trade-off curves for different values of  $\alpha$  for the 11-pulse/19-waveguide system with  $\gamma = 0.83M$ . Points marked (A) represent the best design (by visual inspection) given any trade-off curve due to its minimum  $\delta_{FT}$ .

Figure 9: Output temporal profiles corresponding to points marked (A) (representing designs with the flattest tops) on the trade-off curves in Fig. 8 for (a)  $\alpha = 0.45$ ; (b)  $\alpha = 0.35$ ; (c)  $\alpha = 0.25$ ; and (d)  $\alpha = 0.15$ .

Figure 10: Examples of normalized single-pass spectral profiles ( $\log\left(\frac{|H(j\omega)|^2}{|H_0(j\omega_0)|^2}\right)$ ) for various values of  $\alpha$ . (a)  $\alpha = 0.45$ ; (b)  $\alpha = 0.35$ ; (c)  $\alpha = 0.25$ ; and (d)  $\alpha = 0.15$ , along with the unperturbed spectrum, corresponding to the time domain plots from Fig. 10.

Figure 11: Optimum Trade-off curves for (39 waveguide/19 pulse) and (19 waveguide/11 pulse) systems. The Point marked (A) on both curves correspond to optimal designs (i.e.,  $\gamma = 0.83M$  and  $\alpha = 0.45$ ).

Figure 12: Output temporal intensity profiles corresponding to optimum designs (corresponding to points marked (A) in Fig. 11) for 11 and 19 output pulses systems respectively. For generating the time domain plots, we have used design 1 from Tables II and III.

Figure 13: Output temporal profiles for various pathlength jitters for the 11 pulse system: (a) the unperturbed output (b)  $\pm\frac{\lambda}{100}$  (c)  $\pm\frac{\lambda}{50}$  (d)  $\pm\frac{\lambda}{25}$  (e)  $\pm\frac{\lambda}{16}$  (f)  $\pm\frac{\lambda}{8}$ .

Figure 14: Comparison of single-pass spectral profiles between the unperturbed design and with  $\pm\frac{\lambda}{8}$  jitter.

### Table captions

Table I Table of Range of calculated excess insertion loss for designs with the flattest top (points marked (A) on all trade-off curves shown in Fig. 8(a) and (b)) for various values of  $\alpha$  for the 11-pulse/19-waveguide system.

Table II Table of Phase v/s Waveguide number for two optimal designs for 11-pulse/19-waveguide system.

Table III Table of Phase v/s Waveguide number for two optimal designs for 19-pulse/39-waveguide system

Table IV Summary of performance of optimal designs (designs 1 and 2 from Tables II and III) for 11-pulse/19-waveguide and 19-pulse/39-waveguide systems

Table V Table of pulse train flatness to pulse train cutoff for different extents of pathlength jitters (worst case).

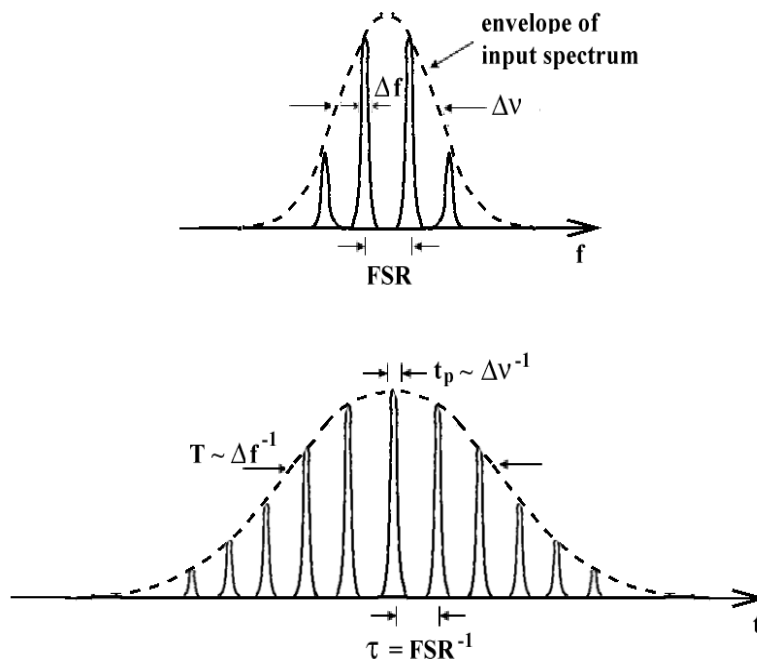


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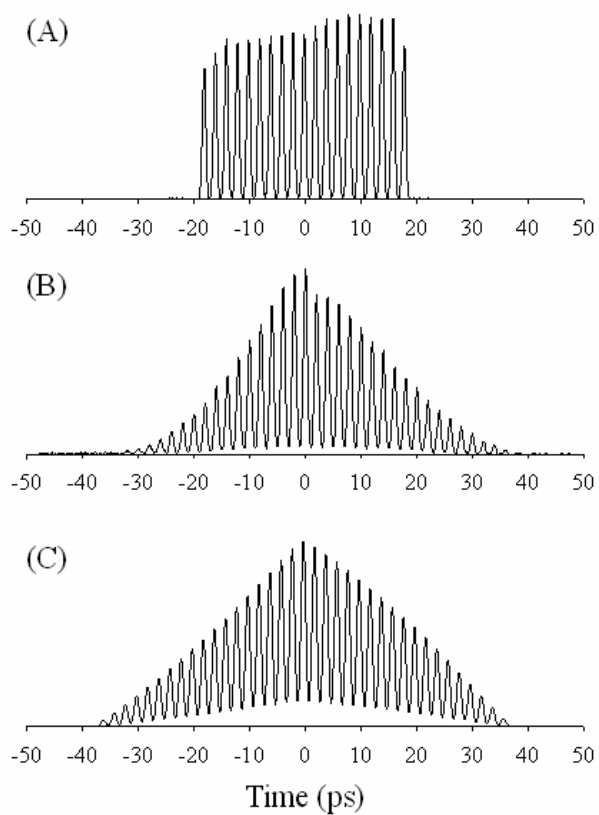


Fig. 2. Output pulse trains. A) Single pass through the AWG. B) Double pass. C) Calculated output pulse train obtained by auto-convolving the single pass temporal response. (from Leaird et.al [8]).

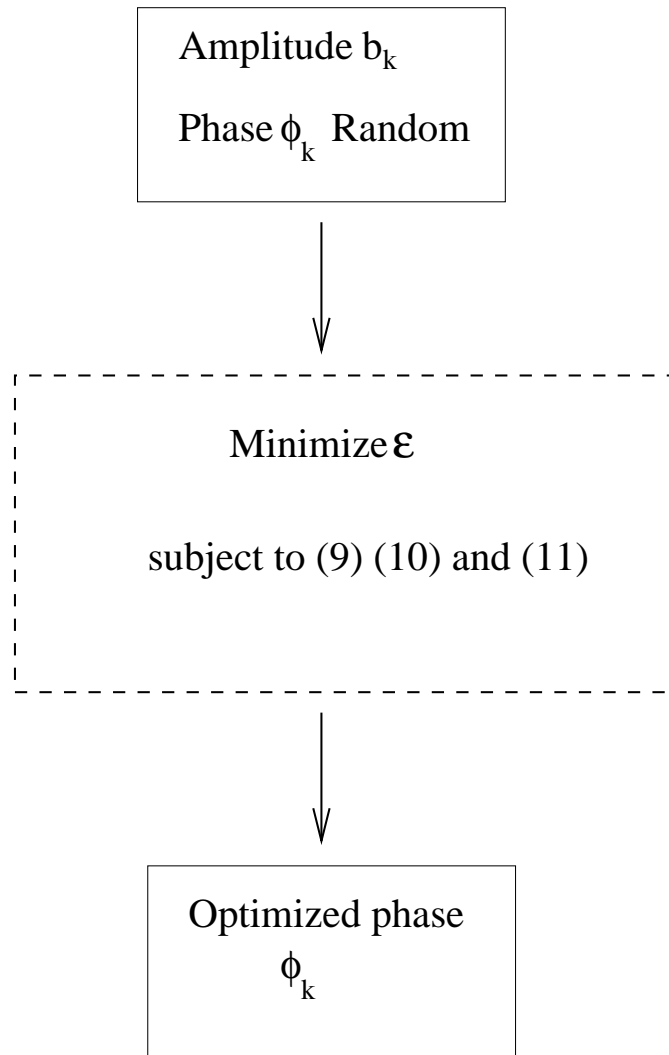


Fig. 3. Block diagram of the optimization procedure. Using the amplitude profile  $b_k$  and the optimized phase profile  $\phi_k$ , output target waveform, and spectral properties are evaluated via (8) and (15).

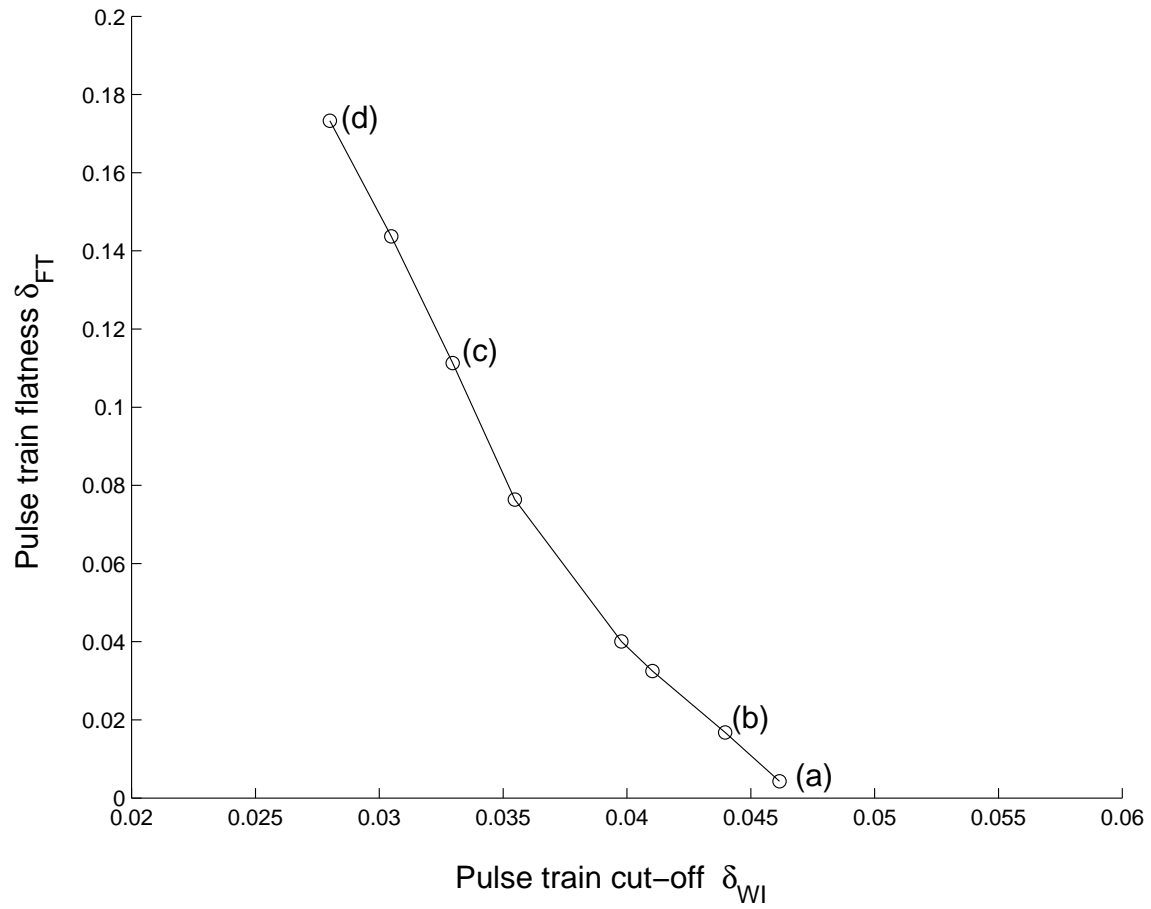


Fig. 4. A sample trade-off curve. Here, we have set  $\gamma = 0.83M$  and  $\alpha = 0.45$ . We obtain the various points by setting  $w_{WI} = 1$  and varying  $w_{FT}$  gradually. Points marked (a) through (d) correspond to  $w_{FT} = 0.01, 0.04, 0.4,$  and  $0.7$  respectively.

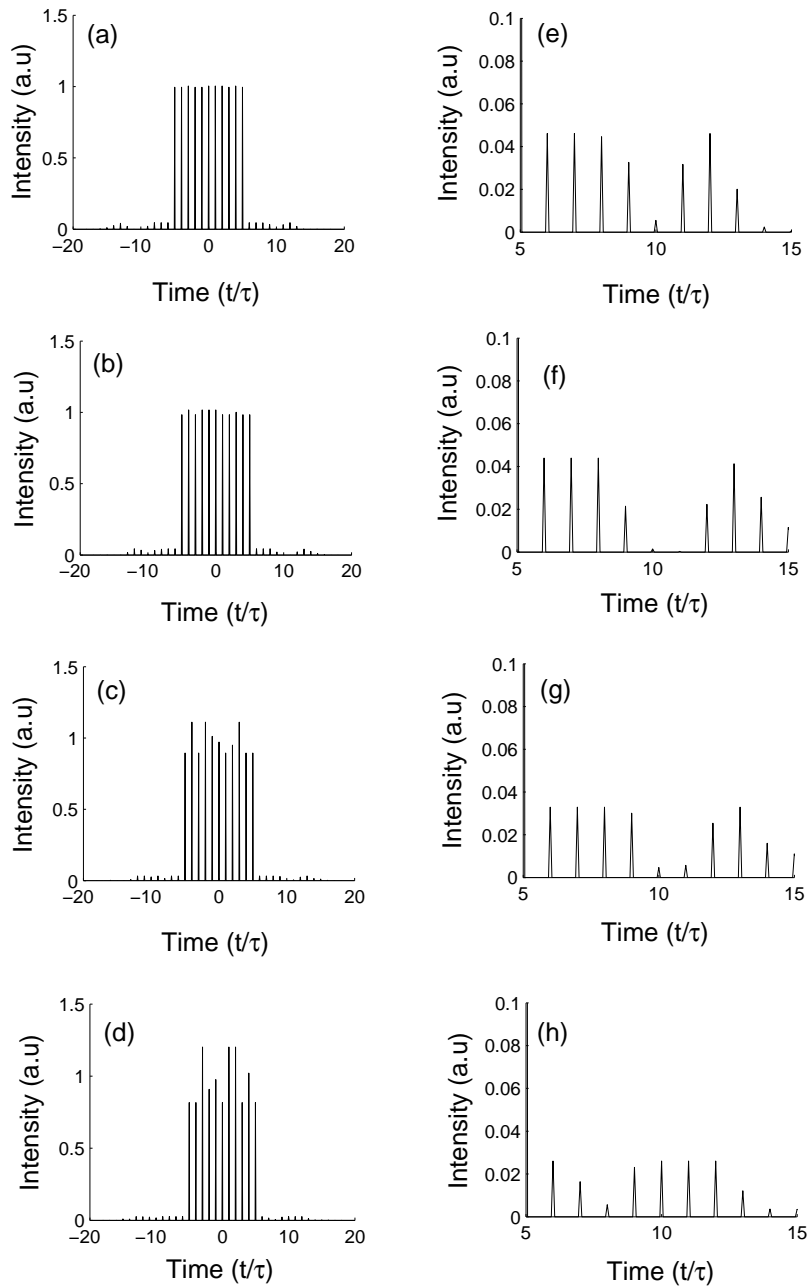


Fig. 5. Output temporal intensity profiles corresponding to points (a), (b), (c) and (d) on the trade-off curve. Figures (e), (f), (g) and (h) represent a closer look at the wing intensity corresponding to (a), (b), (c) and (d) respectively.

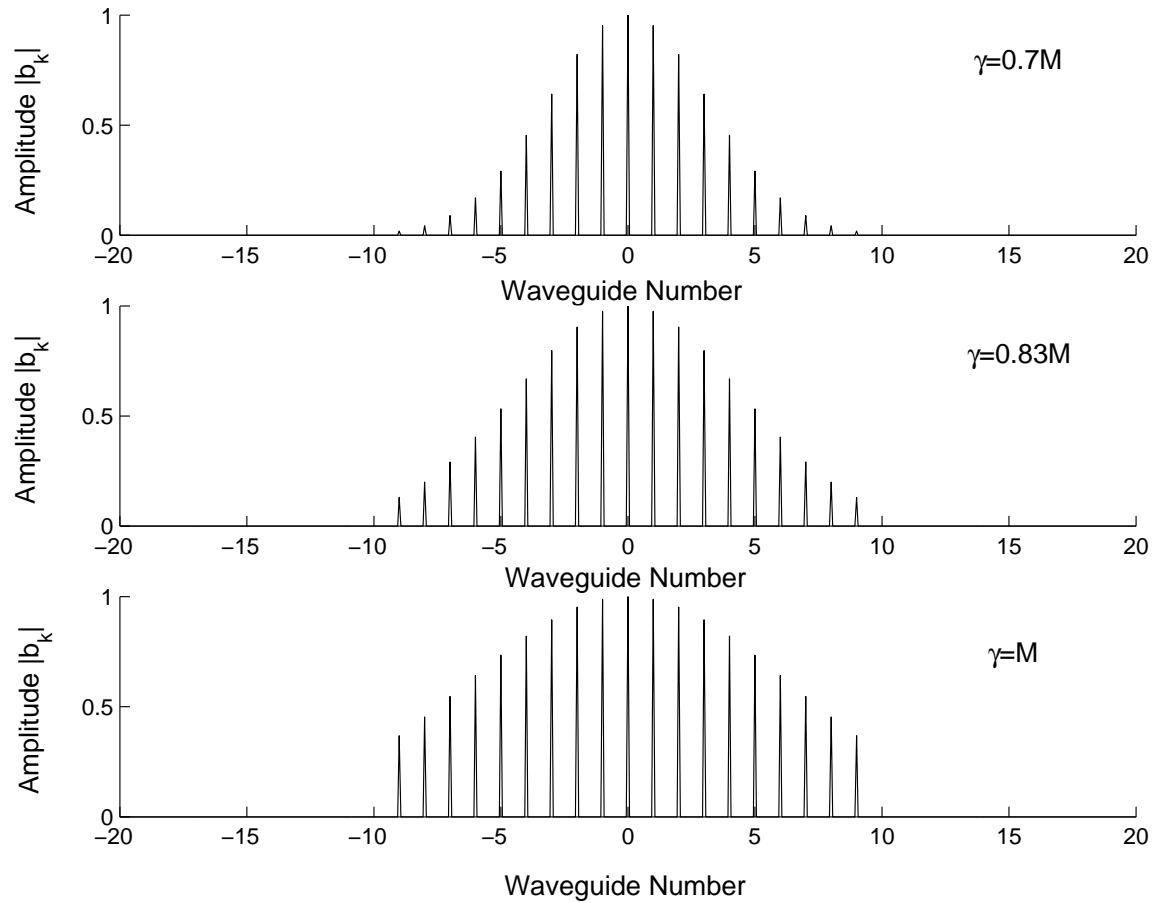


Fig. 6. Excitation Profile  $|b_k|$  for (a)  $\gamma = 0.7M$  (b)  $\gamma = 0.83M$  (c)  $\gamma = M$  for the 19-waveguide system.

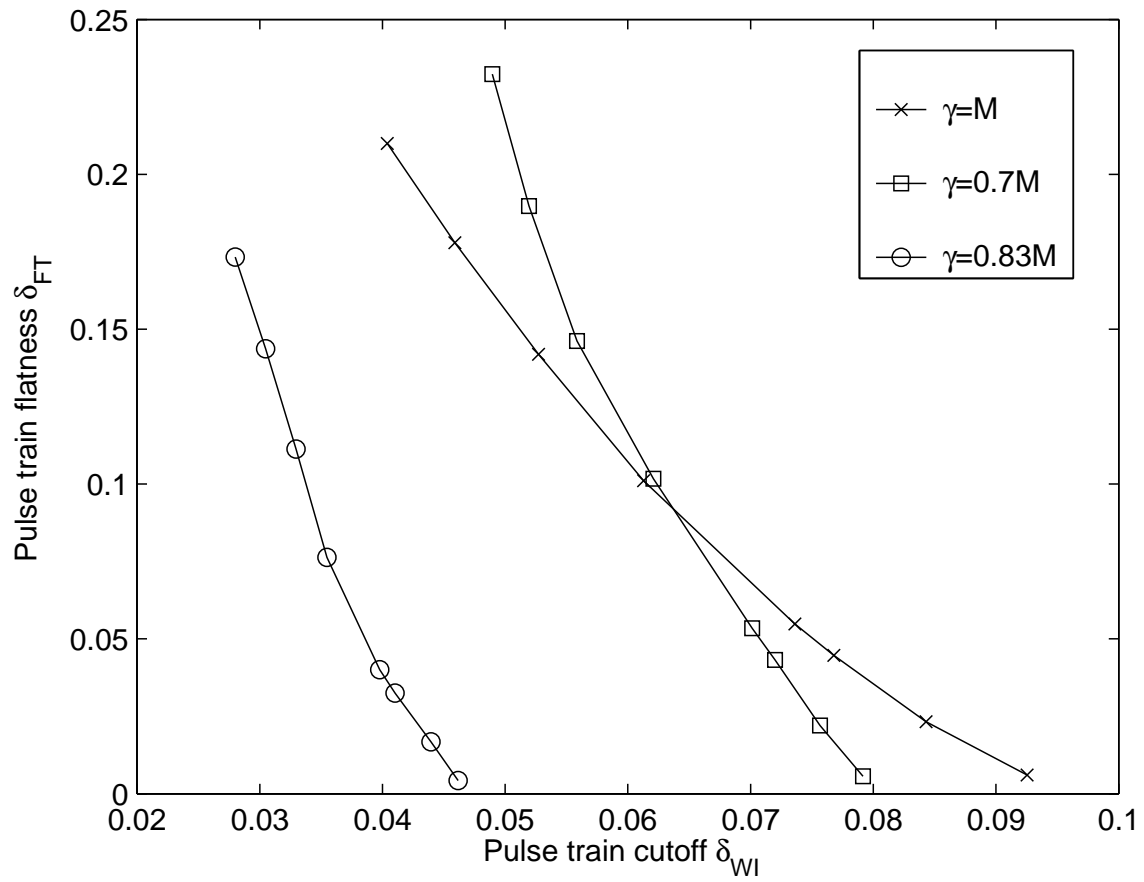


Fig. 7. Trade-off curve for (a)  $\gamma = 0.7M$  (b)  $\gamma = 0.83M$  (c)  $\gamma = M$  for the 11-pulse/19-waveguide system with  $\alpha = 0.45$ .

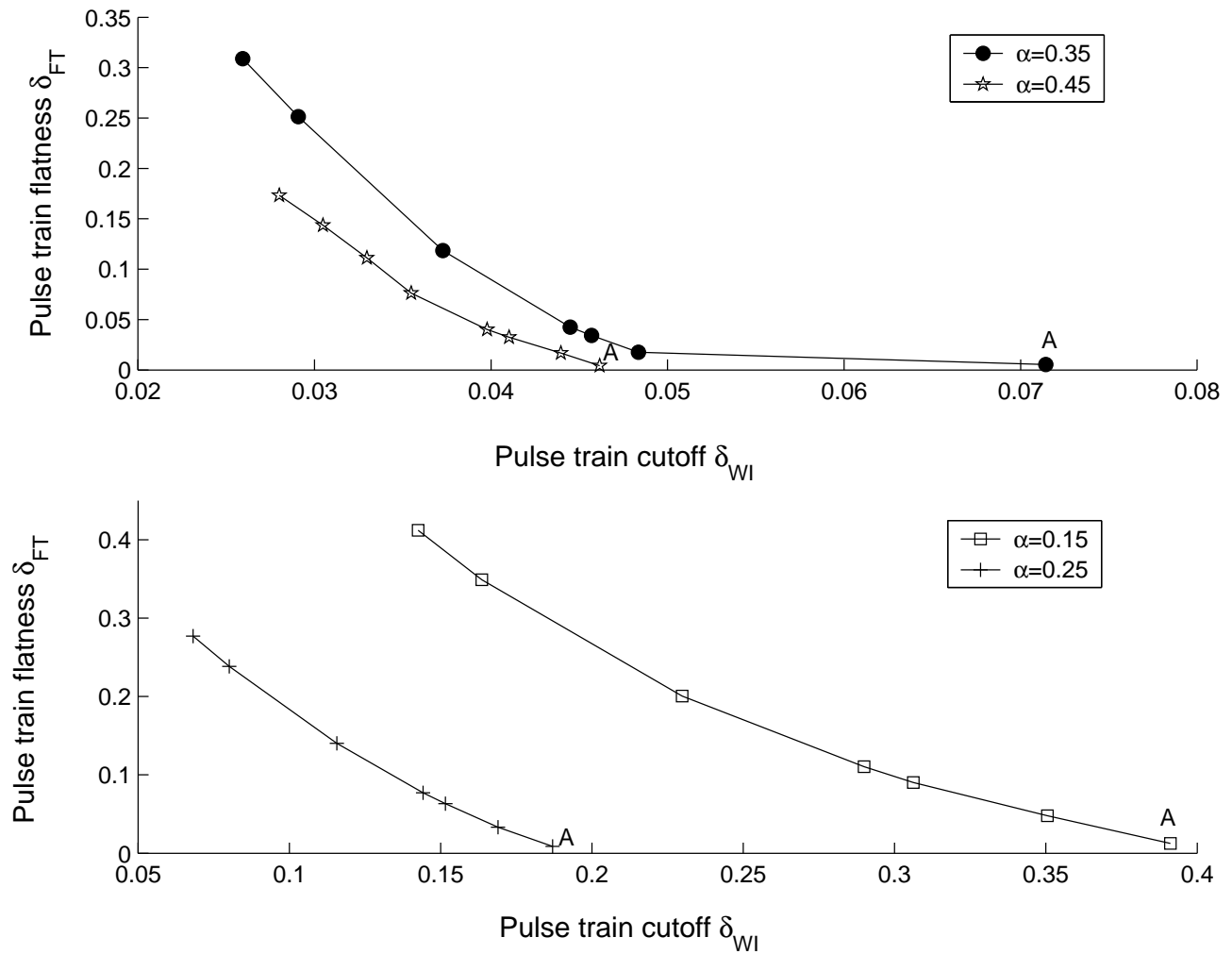


Fig. 8. Trade-off curves for different values of  $\alpha$  for the 11-pulse/19-waveguide system with  $\gamma = 0.83M$ . Points marked (A) represent the best design (by visual inspection) given any trade-off curve due to its minimum  $\delta_{FT}$ .

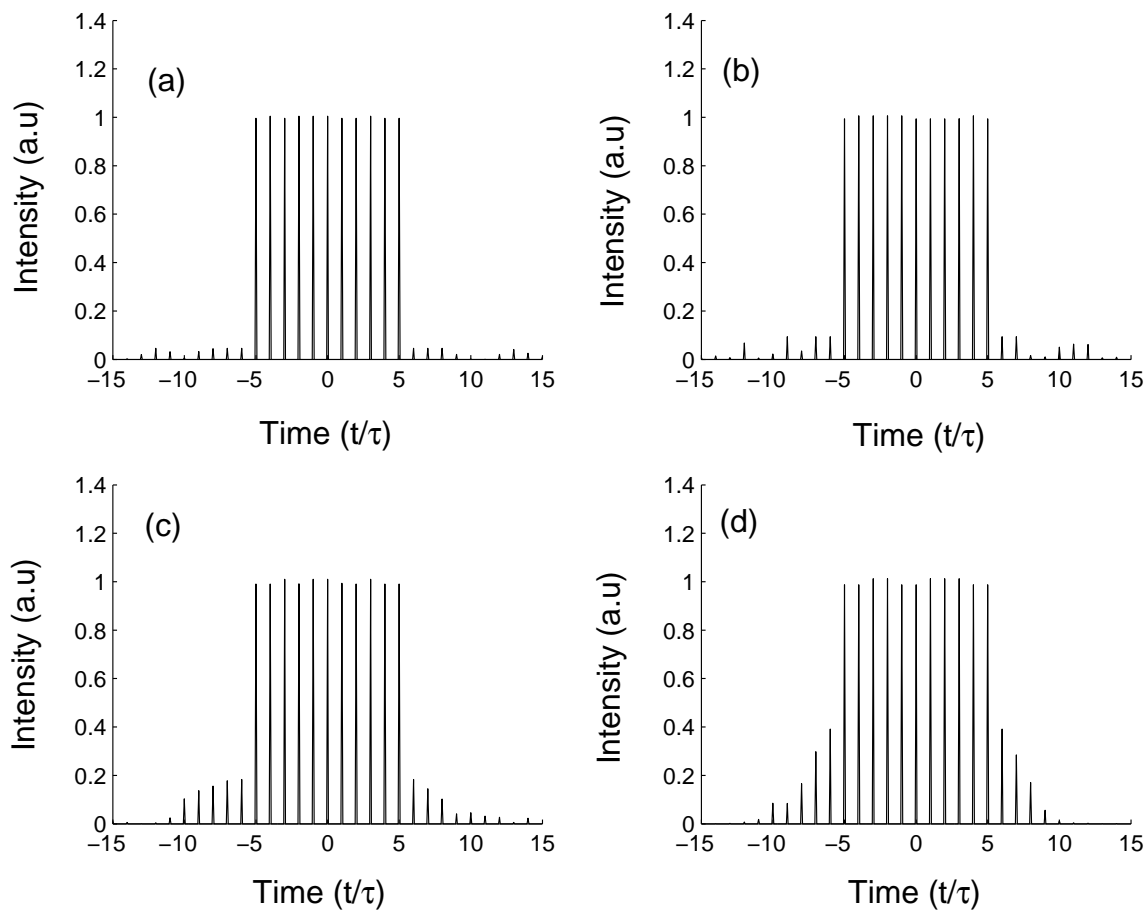


Fig. 9. Output temporal profiles corresponding to points marked (A) (representing designs with the flattest tops) on the trade-off curves in Fig. 8 for (a)  $\alpha = 0.45$ ; (b)  $\alpha = 0.35$ ; (c)  $\alpha = 0.25$ ; and (d)  $\alpha = 0.15$ .

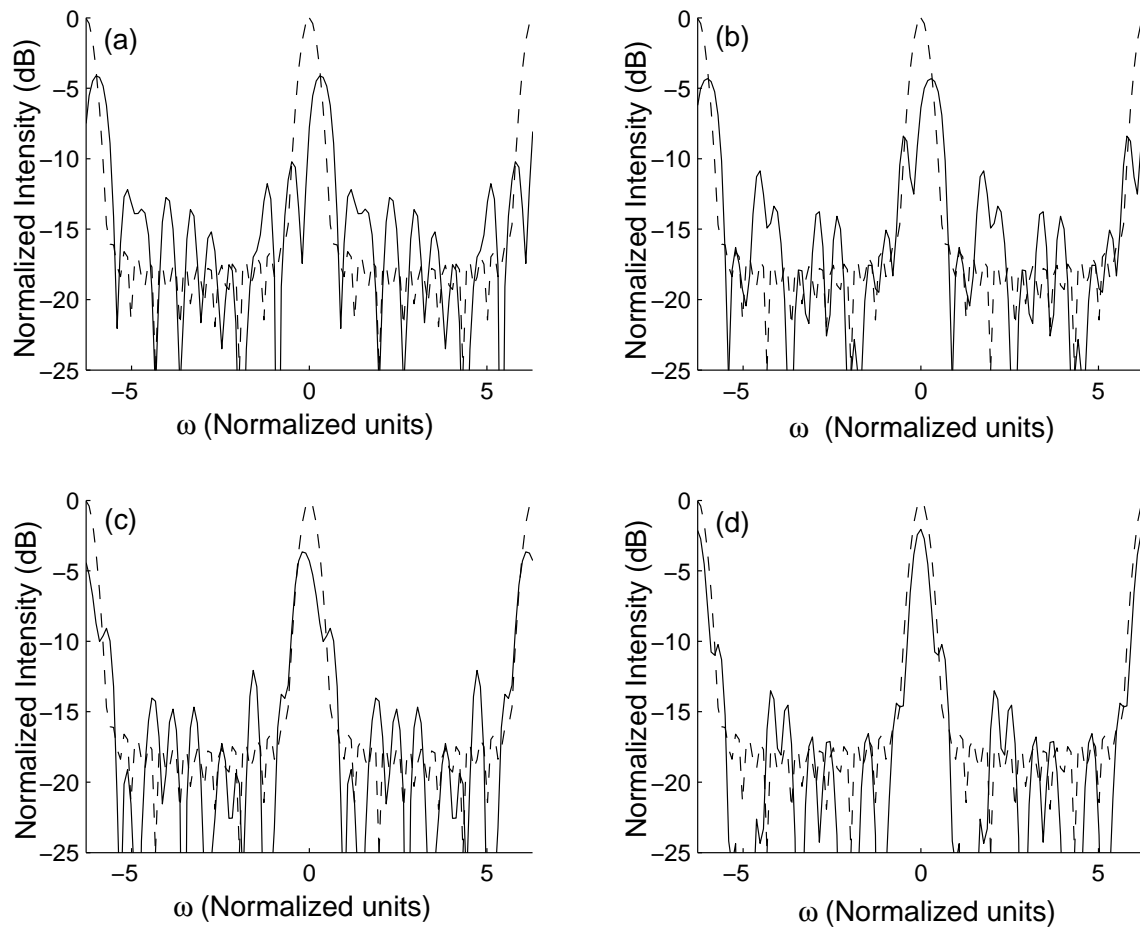


Fig. 10. Examples of normalized single-pass spectral profiles ( $\log \left( \frac{|H(j\omega)|^2}{|H_0(j\omega_0)|^2} \right)$ ) for various values of  $\alpha$ . (a)  $\alpha = 0.45$ ; (b)  $\alpha = 0.35$ ; (c)  $\alpha = 0.25$ ; and (d)  $\alpha = 0.15$ , along with unperturbed spectrum, corresponding to the time domain plots from Fig. 10

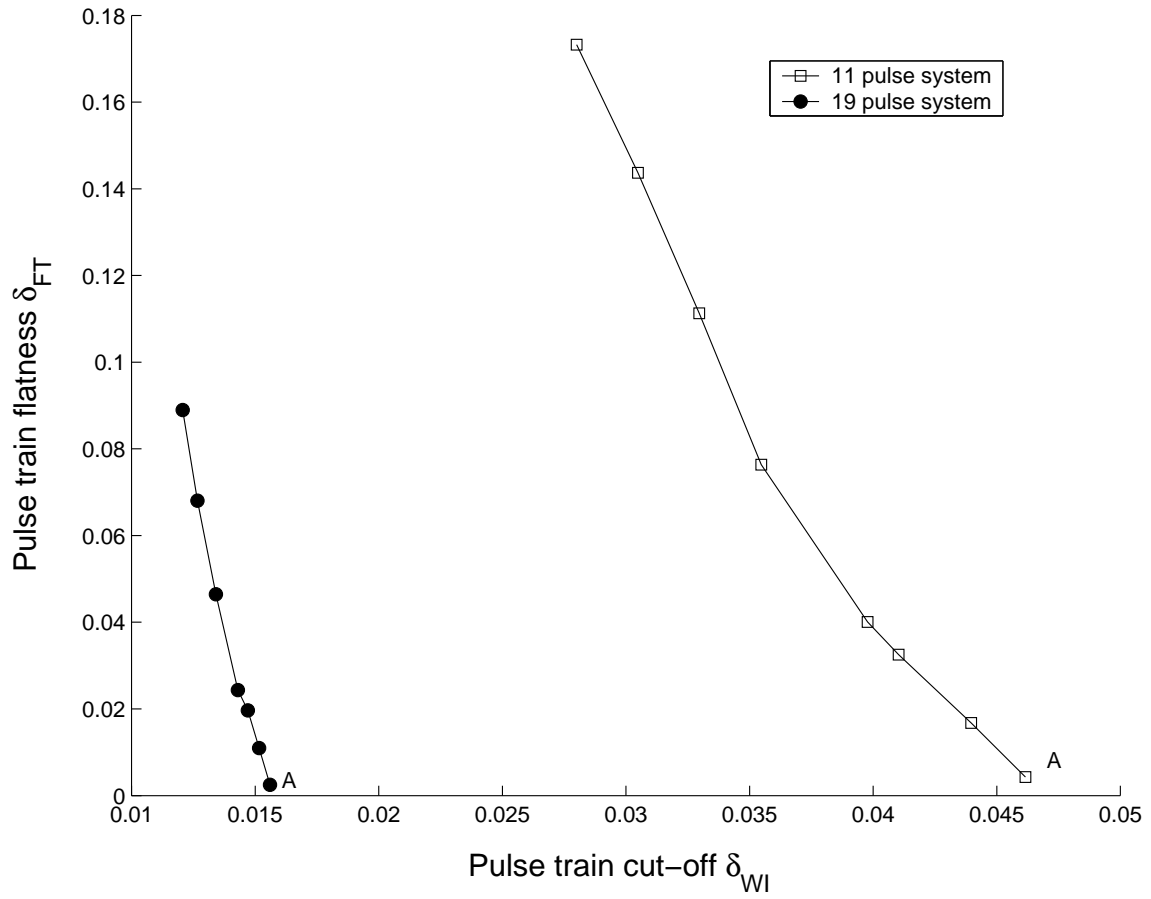


Fig. 11. Optimum Trade-off curves for (39 waveguide/19 pulse) and (19 waveguide/11 pulse) systems. The Point marked (A) on both curves correspond to optimal designs (i.e.,  $\gamma = 0.83M$  and  $\alpha = 0.45$ ).

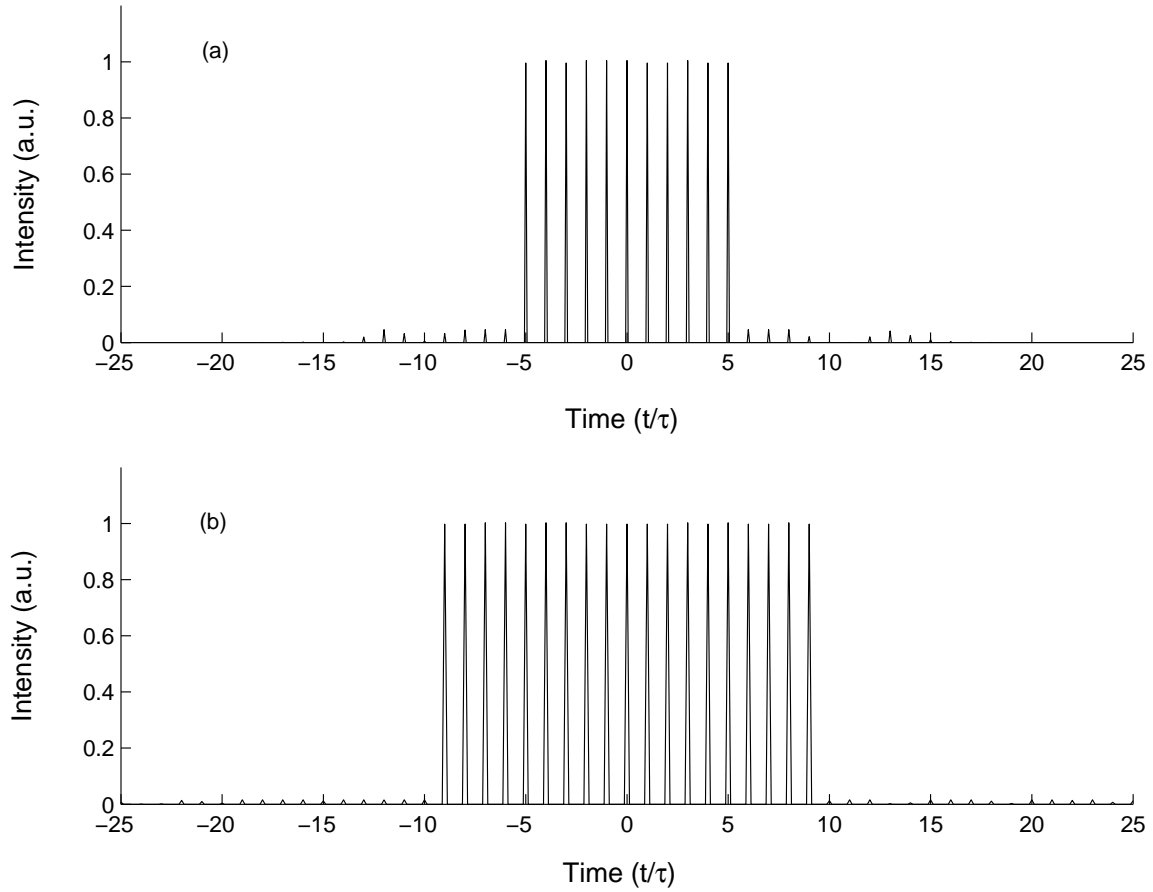


Fig. 12. Output temporal intensity profiles corresponding to optimum designs (corresponding to points marked (A) in Fig. 11) for 11 and 19 output pulses systems respectively. For generating the time domain plots, we have used design 1 from Tables II and III.

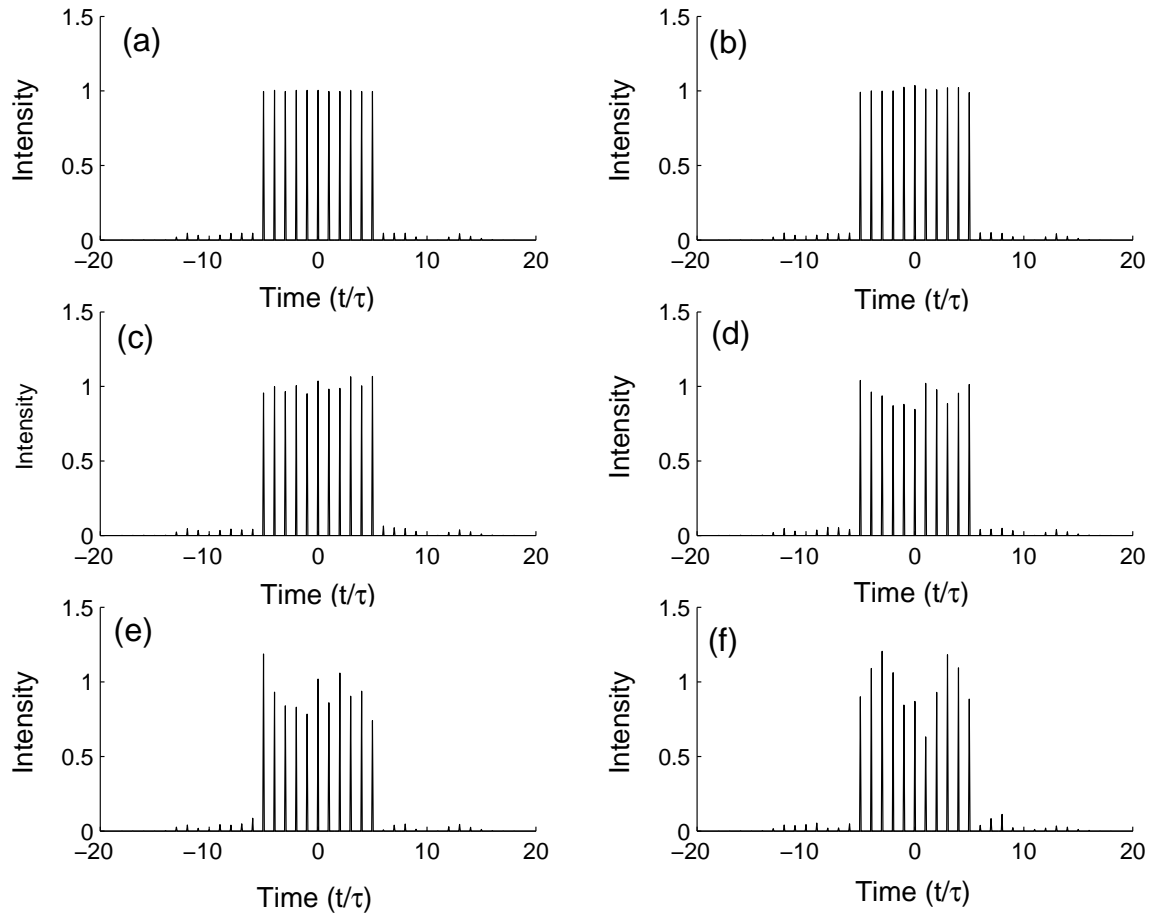


Fig. 13. Output temporal profiles for various pathlength jitters for the 11 pulse system: (a) the unperturbed output (b)  $\pm \frac{\lambda}{100}$  (c)  $\pm \frac{\lambda}{50}$  (d)  $\pm \frac{\lambda}{25}$  (e)  $\pm \frac{\lambda}{16}$  (f)  $\pm \frac{\lambda}{8}$

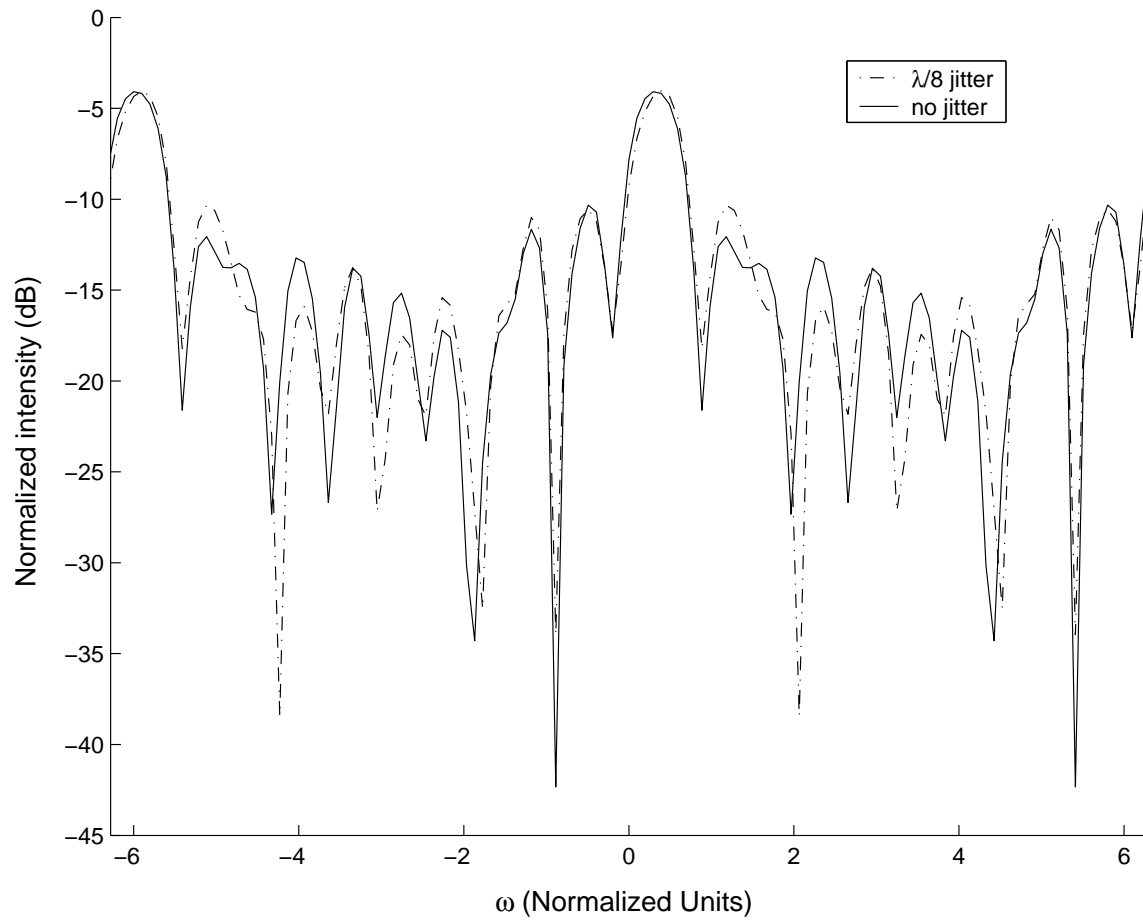


Fig. 14. Comparison of single-pass spectral profiles between the unperturbed design and with  $\pm \frac{\lambda}{8}$  jitter.