EDITORIAL

A number of advances over the past two decades have resulted in numerical solution methods becoming more relevant for problems arising in systems and control: The first is the continuing growth in computing power; the second, recent breakthroughs in optimization theory and algorithms, especially convex optimization; and the third, recent advances in numerical linear algebra. As a consequence, we can today solve using numerical methods, within reasonable time, a number of problems in systems and control for which no analytical solutions are known or likely to exist. This is especially true with regard to convex optimization methods; given the current state of knowledge, problems that reduce to finite-dimensional convex optimization problems are in principle no harder to solve than a system of simultaneous linear equations of the same dimensions. Optimization problems involving Linear Matrix Inequalities, whose application to control is the focus of this special issue, constitute a special yet very wide class of convex optimization problems that has attracted considerable attention from optimization theorists and control researchers alike.

A linear matrix inequality or LMI is a matrix inequality of the form

$$F(\zeta) \triangleq F_0 + \sum_{i=1}^{m} \zeta_i F_i > 0, \tag{1}$$

where $\zeta \in \mathbf{R}^m$ is the variable, and $F_i = F_i^T \in \mathbf{R}^{n\times n}, i = 0, \ldots, m$ are given. The inequality symbol in (1) means that $F(\zeta)$ is positive-definite, i.e., $u^T F(\zeta) u > 0$ for all nonzero $u \in \mathbf{R}^n$. The set $\{\zeta \mid F(\zeta) > 0\}$ is convex. Optimization problems can be then defined with the LMI (1) as their centerpiece. Several software packages for solving LMI optimization problems are currently available[1–3]. The relevance of LMIs to control theory stems from the fact that several important problems from control theory can be reformulated as LMI optimization problems; we refer the reader to the book by Boyd, *et al.*[4] for an introduction to LMI optimization, as well as a comprehensive catalog of problems from system and control theory that can be solved using LMI methods.

The purpose of this special issue is to collect recent advances in the use of LMI techniques for the solution of problems in control theory and applications. An initial call for papers for the special issue was sent out in late 1994, and 20 manuscripts were received in response; we believe that the large number of papers received reflects the great interest that this area enjoys from the control community. The large number of papers that were found acceptable for publication by the reviewers has necessitated splitting the special issue into two parts.

The first part consists of five papers, whose orientation is mostly theoretical:

1. *A linear matrix inequality approach to peak-to-peak gain minimization,* by J. Abedor, K. Nagpal and K. Poolla. A new approach to the problem of peak-to-peak gain minimization ($\mathbf{L}_1$ problem) is presented, with the goal of avoiding the complexity issues associated with existing approaches. By considering an upper bound on the peak gain, controllers are derived that have the same order as the original plant, as in the case of LQG or $\mathbf{H}_\infty$ design.

2. *Mixed* $\mathbf{H}_2/\mathbf{H}_\infty$ *control for time-varying and linear parametrically-varying systems*, by C. Scherer. A solution to the mixed $\mathbf{H}_2/\mathbf{H}_\infty$ control problem of time-varying systems with reduced order controller is provided in terms of differential linear matrix inequalities and rank conditions. This approach is extended to account for time-varying systems. As a special case, a complete characterization of the $\mathbf{H}_2$ problem in terms of LMIs is obtained.

3. *Robust stability analysis using LMIs: Beyond small gain and passivity*, by S. Gupta. LMI indicators for gain boundedness and passivity are extended to handle a wide variety of sector-bounded systems. A numerical convex optimization procedure is proposed to compute the tightest sector bound to which a given system belongs.

4. *Systems with uncertain parameters—Time-variations with bounded derivatives*, by U. Jönsson and A. Rantzer. The framework of integral quadratic constraints is used to exploit derivative bounds on uncertain time-varying parameters for robust stability analysis of linear systems. The results generalize well-known stability results. The stability analysis involves a search for suitable multipliers and can be formulated as a convex optimization problem involving LMIs.

5. *Induced* L_2*-norm control for LPV systems with bounded parameter variation rates,* by F. Wu, X. H. Yang, A. Packard and G. Becker. The problem of stabilizing a parameter-dependent system using a parameter-dependent controller with disturbance/error attenuation requirements (as measured in induced $\mathbf{L}_2$ norm) is considered. The parameters as well as their (bounded) rates of variation are assumed to be known in real time. The approach consists of a bounding technique based on a parameter-dependent Lyapunov function, which results in a semi-infinite dimensional convex optimization problem. Finite dimensional approximations that yield sufficient conditions for successful controller design are then proposed.

The second part of this special issue includes a number of applications that build on theoretical developments. In particular, three of the articles in this part deal extensively with applications, which we believe goes a long way towards demonstrating the practical usefulness of LMI methods; this is essential for the sustained interest of the control community in LMI techniques. The contents of the second part are as follows:

1. L_∞ *norm simultaneous system approximation,* by D. Kavranoglu, M. Bettayeb and M. F. Anjum. A number of LMI-based techniques for optimal system approximation by a lower-order system using the $\mathbf{L}_\infty$ metric are presented. Numerical examples are provided.
2. *Robust state-feedback stabilization of jump linear systems via LMIs*, by L. El Ghaoui and M. Ait-Rami. An LMI-based approach towards the stabilization of linear systems whose dynamics are subject to Markovian jumps is presented. The optimal quadratic control problem for such systems is shown to be solvable via convex optimization.
3. *Linear parameter-varying control of a ducted fan engine*, by B. Bodenheimer, P. Bendotti and M. Kantner. Control techniques designed for parameter-dependent systems are applied to a vectored-thrust, ducted fan engine. The LMI-based controller synthesis technique is shown to perform very well, even though the plant's dynamics can vary

widely across the flight envelope. The approach compares favorably with a classical $\mathbf{H}_\infty$ design.

4. *Analysis and synthesis tools for a class of actuator-limited multivariable control systems: A linear matrix inequality approach*, by V. R. Marcopoli and S. M. Phillips. Actuator limits can introduce severe performance degradation and possible instability of control systems. A methodology to design limit protections for a class of actuator-limited, multivariable control systems is proposed. Examples that illustrate the potential practical impact of the design methodology are provided.
5. *Sampled data model validation: An algorithm and experimental application*, by G. Dullerud and R. Smith. The problem of model validation using experimental data is considered. Specifically, it is shown that continuous-time model validation using discrete data sets is tractable using LMIs. The results are applied to a heating experiment.
6. *Robust synthesis via bilinear matrix inequalities*, by K.-C. Goh, M. G. Safonov and J. H. Ly. While many control problems have been shown to be solvable using LMIs and convex optimization, there are many other control problems of significant engineering importance which will probably never be solvable as convex optimization problems. Such problems include robust controller synthesis. The robust controller synthesis problem with realistic controller constraints, including order and structure constraints, is shown to be equivalent to an optimization problem involving bilinear matrix inequalities.

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[2] L. El Ghaoui, F. Delebecque, and R. Nikoukhah. *LMITOOL: A User-friendly Interface for LMI Optimization.* ENSTA/INRIA, 1995. Software available via anonymous FTP from `ftp.inria.fr`, under directory `pub/elghaoui/lmitool`.

[3] S. Boyd and S. Wu. *SDPSOL: A Parser/Solver for Semidefinite Programs with Matrix Structure. User's Guide, Version Alpha.* Stanford University, March 1995.

[4] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*, volume 15 of *Studies in Applied Mathematics*. SIAM, Philadelphia, PA, June 1994.