Robust Performance Bounds based on Lyapunov Functions for Uncertain Systems

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Abstract

The robust stability analysis of uncertain systems, with various assumptions on the nature of the uncertainties (sector-bounded nonlinear, linear time-invariant, parametric, etc.), as well as their structure (diagonal, block-diagonal, etc.), can be performed in a unified manner using multiplier theory and LMI-based convex optimization. The multipliers used in the stability analysis can be shown to yield a convex parametrization of a subset of Lyapunov functions that provide a certificate of robust stability. We show how these Lyapunov functions can in turn be used to derive bounds on various robust performance measures of the system. We illustrate our approach with three specific robust performance analysis problems.

1 Introduction

Most control system models in use today explicitly incorporate in them "uncertainties" or "perturbations". These uncertainties may model a number of factors, including: dynamics that are neglected to make the model tractable, as with large scale structures; nonlinearities that are either hard to model or too complicated; and parameters that are not known exactly, either because they are hard to measure or because of varying manufacturing conditions. A widely-used model for uncertain systems, shown in Fig. 1, consists of a nominal finite-dimensional, linear, time-invariant system, with the perturbation or uncertainty Δ in the feedback loop. The signal w represents exogenous inputs, and z represents all outputs of interest. Often additional information about Δ is either known or assumed; common examples are that Δ is diagonal or block-diagonal; sector-bounded memoryless, linear time-invariant or parametric; bounded in norm, passive, etc. The analysis of and design for such control system models is usually referred to as "robust control with structured perturbations".

One of the most fundamental questions concerning the system in Fig. 1 is that of stability: "Is the model stable irrespective of the perturbation Δ, that is, do all solutions of the system equations go to zero, irrespective of Δ?" This is also referred to as the robust stability problem. Some of the approaches for solving this problem, with various assumptions on Δ, are the use of the small-gain, passivity or circle-criteria [9], the Popov criterion[17], and μ or K_m analysis methods [10, 19, 7].

Robust stability is but one desired feature of an uncertain system; of considerable importance are questions beyond stability, known broadly as *robust performance* problems. Robust performance analysis problems concern the computation of the worst possible value, over all uncertainties, of performance indices; these performance indices may be bounds on some state variables, norms of the map from w to z etc. An example of a robust performance analysis problem is the so-called $\mathbf{H}_2$ problem, which is the computation of the largest possible RMS value of the output z over all Δ, when the input w is unit-intensity white noise. This finds application where the average value of a certain signal is of interest, when the system is affected by an unpredictable input that is modeled as white noise. Another example is the computation of the largest possible RMS gain, over all uncertainties, from w to z; this is also known as the $\mathbf{H}_\infty$ performance analysis problem.

It has been found recently that several stability analysis methods for control systems can be unified in the setting of integral quadratic constraints with multipliers [18, 5, 16, 20, 14, 15]. As a consequence, several sufficient conditions for robust stability can be performed without frequency sampling, using convex optimization techniques based on linear matrix inequalities [6, 5]. Moreover, multiplier or IQC-based techniques yield a *convex* parametrization of a subset of Lyapunov functions that prove robust stability [4]; this is of central importance to this paper. By imposing additional conditions on these Lyapunov functions, we will derive bounds on robust performance (see [8] for numerous examples) of the system in Fig. 1. The "best" performance bounds can then obtained by numerically optimizing over the subset of Lyapunov functions that prove robust performance. In §2, we describe the general idea behind the approach; we also address here three specific robust performance analysis problems using this general approach. In §3, we present a simple numerical example, and conclude with §4.

2 Robust performance bounds from Lyapunov functions

Consider the uncertain system shown in Fig. 1, consisting of a stable finite-dimensional linear system with an uncertainty or perturbation Δ in the feedback loop:

$$\frac{d}{dt}x(t) = Ax(t) + B_p p(t) + B_w w(t), \quad q(t) = C_q x(t) + D_{qp} p(t), \quad z(t) = C_z x(t), \tag{1a}$$

$$p(t) = -\Delta(q, t). \tag{1b}$$

where $x(t) \in \mathbf{R}^n$, $p(t) \in \mathbf{R}^{n_p}$, $q(t) \in \mathbf{R}^{n_q}$, $w(t) \in \mathbf{R}^{n_w}$ and $z(t) \in \mathbf{R}^{n_z}$. (For convenience, we have assumed that there is no feed-through from w to z, w to q and p to z.) We shall assume that $n_p = n_q = m$; with little difficulty, the results herein can be extended to the more general case $n_p \neq n_q$. For future reference, we let $G(s)$ denote the transfer function of the linear part of the system, from $[w^T \quad p^T]^T$ to $[z^T \quad q^T]^T$. Equations (1) can be interpreted as representing a family of systems, each member corresponding to some Δ. The perturbation Δ is in general nonlinear. Additional information about the structure (diagonal, block-diagonal etc) and nature (nonlinear bounded, sector-bounded, LTI, parametric, etc) is usually available.

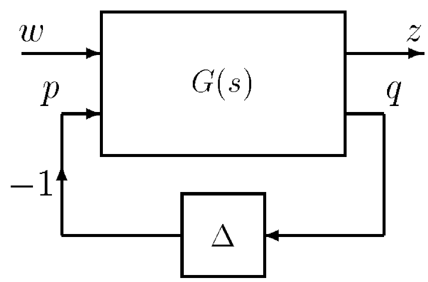

Figure 1: Robustness analysis framework.

Suppose there exists a positive-definite function[1] $V : \mathbf{R}^n \times \mathbf{R}_+ \to \mathbf{R}_+$ such that $dV(x,t)/dt < 0$ along the trajectories of (1). Then from standard arguments from Lyapunov theory, the system (1) is robustly stable, i.e., for every member of the family and for every initial condition, every solution $x(t)$ of (1) satisfies $\lim_{t\to\infty} x(t) = 0$.

Additional conditions on the Lyapunov function can be imposed, yielding robust performance bounds as follows (see for example [3, §6] and [8]): For a given initial condition $x(0)$ for the linear part (1a), suppose that the Lyapunov function V satisfies

$$\frac{d}{dt}V(x,t) < -\phi_\alpha(x(t), w(t), z(t)) \text{ along the trajectories of (1).} \tag{2}$$

Then it is a simple exercise to show that

$$V(x(0),0) > \int_0^\infty \phi_\alpha(x(t), w(t), z(t))\, dt, \tag{3}$$

or we have an upper bound on the performance index

$$\mathcal{P}_\alpha \triangleq \int_0^\infty \phi_\alpha(x(t), w(t), z(t))\, dt. \tag{4}$$

Thus, if we can find a Lyapunov function V satisfying (2), then we can obtain upper bounds on robust performance indices of the form (4), for system (1). Natural questions then are:

1. What is the form of these Lyapunov functions (quadratic function of $x(t)$, some functional of $x(t)$, etc)?
2. How can we optimize over the set of such Lyapunov functions, in order to get the best bounds on robust performance?

The answer to the first question, as we will see through the examples presented in §2.1–§2.3, depends on the structure and nature of Δ. A partial answer to the second question is provided by numerical optimization based on linear matrix inequalities (LMIs)[2]. In particular, we will show that in several cases, a rich subset of Lyapunov functions that satisfy condition (2) can be parametrized using LMIs, and that the objective $V(x(0),0)$ is a convex function of the parameters. Therefore performance bounds for the system (1) can be very efficiently computed using standard software for numerical LMI optimization [12, 13, 25].

[1] For precise technical definitions, mathematical preliminaries, and the notation used here, see [9] and [21].

[2] We refer the reader to the book [8] for an introduction to the application of LMI optimization to problems from system and control theory.

2.1 Example: Unstructured strictly passive Δ; bound on the output energy

Consider the special case when the uncertainty Δ is nonlinear and strictly passive, i.e., for some real β and $\epsilon > 0$,

$$\int_0^T q(t)^T \Delta(q,t)\, dt \geq \epsilon \int_0^T q(t)^T q(t)\, dt - \beta \text{ for all } T \geq 0 \text{ and all } q \in \mathbf{L}_2. \tag{5}$$

The performance analysis problem considered here is the following:

> With exogenous input $w = 0$, find bounds on the energy in the output, i.e., $\int_0^\infty z(t)^T z(t)\, dt$, for a given initial condition $x(0)$ of the state of the linear part of the system.

In order to derive upper bounds on $\int_0^\infty z(t)^T z(t)\, dt$, we seek Lyapunov functions that satisfy condition (2) with $\phi_\alpha(x, w, z) = z^T z$, i.e.,

$$\frac{d}{dt} V(x,t) < -z(t)^T z(t). \tag{6}$$

Then $V(x(0), 0)$ yields an upper bound on the worst-case output energy.

It turns out (see [22, 23]) that the Lyapunov functions that are appropriate here are of the form

$$V(x,t) = x(t)^T P x(t) - 2\int_0^t p(\tau)^T q(\tau)\, d\tau + 2\beta,$$

where $P > 0$. Condition (6) holds along the trajectories of system (1) if the following condition holds:

$$\begin{bmatrix} A^T P + PA + C_z^T C_z & PB_p - C_q^T \\ B_p^T P - C_q & -(D_{qp} + D_{qp}^T) \end{bmatrix} < 0. \tag{7}$$

Thus, the best upper bound on the worst-case energy of z, for a given initial condition $x(0)$ of the linear part of the system, is simply

$$\begin{array}{ll} \text{minimize} & x(0)^T P x(0) \\ \text{subject to} & P > 0, \text{ and } (7) \end{array}$$

2.2 Example: Diagonal, time-invariant, sector-bounded, memoryless nonlinearities Δ; bound on the state variables

Next, consider the case when: D_{qp} is zero; the uncertainty Δ is diagonal, i.e., $p_i(t) = -\delta_i(q_i(t))$; and each δ_i is a time-invariant, memoryless nonlinearity in sector $[0, \infty)$. For this case, the performance analysis problem considered is:

> With exogenous input $w = 0$, find bounds on the state $x(t)$, $t \geq 0$, for a given initial condition $x(0)$ of the state of the linear part of the system.

In other words, we seek an invariant set for the trajectories $x(t)$ of the linear part of the system. For every positive-definite function V for which $dV(x,t)/dt < 0$ holds along the trajectories of system (1), we have

$$V(x(t),t) \leq V(x(0),0) \text{ for } t \geq 0.$$

This inequality can then be used to derive bounds on $x(t)$.

It turns out (see [9, 8]) that the Lyapunov functions that are appropriate here are of the form

$$V(x,t) = x(t)^T P x(t) + 2\sum_{i=1}^{m} \mu_i \int_0^{q_i(t)} \delta_i(\sigma)\, d\sigma - 2\sum_{i=1}^{m} \lambda_i \int_0^t p_i(\tau) q_i(\tau)\, d\tau,$$

where $P > 0$, $\mu_i \geq 0$ and $\lambda_i \geq 0$. In this case, condition $dV(x,t)/dt < 0$ is equivalent to the LMI with

$$\begin{bmatrix} A^T P + PA & PB_p - (\Lambda C_q + M C_q A)^T \\ B_p^T P - \Lambda C_q - M C_q A & -(M C_q B_p + B_p^T C_q^T M) \end{bmatrix} < 0 \tag{8}$$

where $P > 0$, $M = \mathbf{diag}\,(\mu_1, \ldots, \mu_m) \geq 0$ and $\Lambda = \mathbf{diag}\,(\lambda_1, \ldots, \lambda_m) \geq 0$. If condition (8) holds, then the ellipsoid

$$\mathcal{E} = \left\{ \psi \in \mathbf{R}^n \;\middle|\; \psi^T P \psi \leq x(0)^T P x(0) \right\}$$

is an invariant ellipsoid for the state x.

The problem of finding the smallest invariant ellipsoid (using our techniques), i.e., one with the smallest major axis, can be solved by solving the LMI

$$\begin{array}{ll} \text{maximize} & \mu \\ \text{subject to} & P > \mu I, \;\; x(0)^T P x(0) \leq 1 \text{ and } (8) \end{array}$$

and using the optimal P to construct the invariant ellipsoid.

2.3 Example: Diagonal, constant, real Δ; bound on the largest $\mathbf{H}_2$ norm

We finally consider the case when Δ is a constant, unknown, diagonal matrix, with entries that are nonnegative. The performance analysis problem considered is:

Find the worst-case $\mathbf{H}_2$ norm from w to z over all Δ.

The Lyapunov functions that are appropriate in this case are motivated by multiplier theory [9, 5, 4]. Consider the system shown in Fig. 2, where

1. $W_+(s)$, $W_-(-s)^T$ are diagonal, stable and stably invertible
2. $W_-(-s)^T \Delta W_+(s)^{-1}$ is strictly passive for all Δ (9)

Then, there is a one-to-one correspondence between the state-trajectories of the systems in Figs. 1 and 2. We have translated our knowledge of Δ into extra degrees of

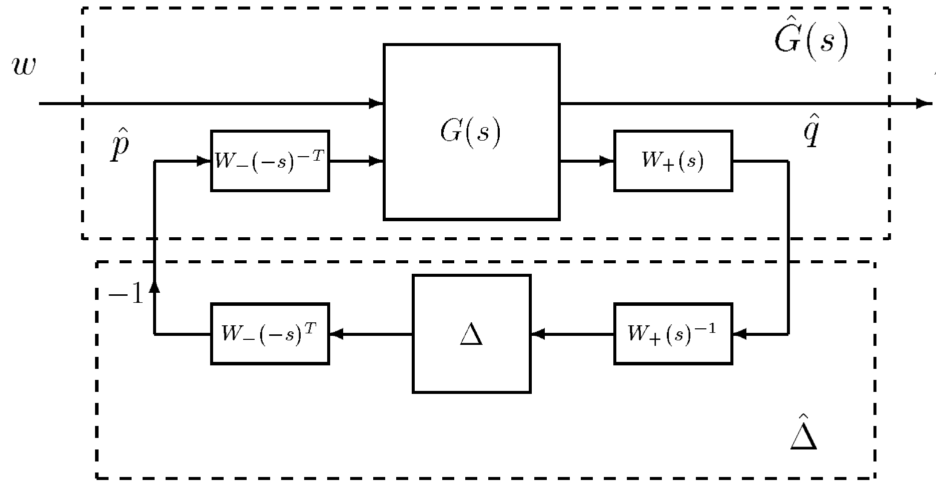

Figure 2: System with multipliers.

freedom, represented by our multipliers W_+ and W_-. Then, we can search for Lyapunov functions for the "augmented" system, which in turn, yields structured Lyapunov functionals for the original system (see [4]).

With $\hat{x}$ denoting the state vector of $\hat{G}$, let the state-equations of the system in Fig. 2 be

$$\frac{d}{dt}\hat{x}(t) = \hat{A}\hat{x}(t) + \hat{B}_p\hat{p}(t) + \hat{B}_w w(t), \quad \hat{q}(t) = \hat{C}_q\hat{x}(t) + \hat{D}_{qp}\hat{p}(t), \quad z(t) = \hat{C}_z\hat{x}(t), \tag{10a}$$

$$\hat{p}(t) = -\hat{\Delta}(\hat{q}(t), t) \tag{10b}$$

Then, a suitable Lyapunov function is

$$V(\hat{x}, t) = \hat{x}(t)^T\hat{P}\hat{x}(t) - 2\int_0^t \hat{p}(\tau)^T\hat{q}(\tau)\, d\tau,$$

where $\hat{P} > 0$.

It can be shown that if this Lyapunov function satisfies

$$\frac{d}{dt}V(\hat{x}, t) < -z(t)^T z(t), \tag{11}$$

then $\sqrt{\mathbf{Tr}\, \hat{B}_w^T\hat{P}\hat{B}_w}$ yields an upper bound on the worst-case $\mathbf{H}_2$ norm from w to z over all Δ.

The problem of computing the best upper bound then is the following:

$$\begin{array}{ll} \text{minimize} & \sqrt{\mathbf{Tr}\, \hat{B}_w^T\hat{P}\hat{B}_w} \\ \text{subject to} & \hat{P} > 0, \quad (9) \text{ and } (11) \end{array}$$

The variables in this optimization problem are $\hat{P}$, and appropriate W_+ and W_-; owing to the latter two variables, it is an infinite-dimensional optimization problem. In practice, it can be approximately solved by restricting W_+ and W_- to lie in a finite-dimensional set.

We will not describe the details here, instead we will merely state the final optimization problem:

Let $W_1, \ldots, W_{r_1+r_2}$, be some strictly proper, stable diagonal $m \times m$ transfer functions. Let $C_-(sI - A_-)^{-1}B_-$ be a minimal state space realization of $\left[W_1(s)^T \ \ldots \ W_{r_1}(s)^T\right]^T$, and let $C_+(sI - A_+)^{-1}B_+$ be a minimal state space realization of $\left[W_{r_1+1}(s)^T \ \ldots \ W_{r_1+r_2}(s)^T\right]^T$. Let

$$\mathbf{\Theta}_{\rm const} = \left\{ \theta \,\middle|\, \theta = \begin{bmatrix} \xi_{11} & \cdots & \xi_{1r_2} & \chi_1 \\ \vdots & \ddots & \vdots & \vdots \\ \xi_{r_1 1} & \cdots & \xi_{r_1 r_2} & \chi_{r_1} \\ \psi_1 & \cdots & \psi_{r_2} & \phi \end{bmatrix}, \quad \xi_{ij}, \psi_i, \chi_j, \phi \in \mathbf{R}^{m\times m} \text{ and diagonal} \right\}.$$

Then an upper bound to the worst-case $\mathbf{H}_2$ norm from w to z for system (1) when $\Delta > 0$ is diagonal and real is given by the square root of the optimal objective of the following LMI problem:

$$\text{Minimize:} \qquad \mathbf{Tr}\, \tilde{B}_w^T \tilde{P} \tilde{B}_w$$

subject to :

$$\begin{bmatrix} A_{\rm W}^T P_{\rm W} + P_{\rm W} A_{\rm W} & P_{\rm W} B_{\rm W} \\ B_{\rm W}^T P_{\rm W} & 0 \end{bmatrix} - \begin{bmatrix} C_{\rm W} & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & \theta_{11}^T & \theta_{21}^T \\ \theta_{11} & 0 & \theta_{12} \\ \theta_{21} & \theta_{12}^T & \theta_{22} + \theta_{22}^T \end{bmatrix} \begin{bmatrix} C_{\rm W} & 0 \\ 0 & I \end{bmatrix} \leq 0$$

$$\begin{bmatrix} \tilde{A}^T \tilde{P} + \tilde{P}\tilde{A} + \tilde{C}_z^T \tilde{C}_z & \tilde{P}\tilde{B}_p \\ \tilde{B}_p^T \tilde{P} & 0 \end{bmatrix} - \begin{bmatrix} \tilde{C}_q & 0 \\ 0 & I \end{bmatrix}^T \begin{bmatrix} 0 & \theta_{11} & \theta_{12} & \theta_{12} D_{qp} \\ \theta_{11}^T & 0 & 0 & \theta_{21}^T \\ \theta_{12}^T & 0 & 0 & \theta_{22}^T \\ D_{qp}^T \theta_{12}^T & \theta_{21} & \theta_{22} & \theta_{22} D_{qp} + D_{qp}^T \theta_{22}^T \end{bmatrix} \begin{bmatrix} \tilde{C}_q & 0 \\ 0 & I \end{bmatrix} \leq 0$$

$$\begin{bmatrix} \theta_{11} & \theta_{12} \\ \theta_{21} & \theta_{22} \end{bmatrix} \in \mathbf{\Theta}_{\rm const}, \quad \tilde{P} > 0, \quad P_{\rm W} > 0$$

where

$$A_{\rm W} = \begin{bmatrix} A_+ & 0 \\ 0 & A_- \end{bmatrix}, \quad B_{\rm W} = \begin{bmatrix} B_+ \\ B_- \end{bmatrix}, \quad C_{\rm W} = \begin{bmatrix} C_+ & 0 \\ 0 & C_- \end{bmatrix},$$

$$\tilde{A} = \begin{bmatrix} A_- & 0 & 0 \\ 0 & A_+ & B_+ C_q \\ 0 & 0 & A \end{bmatrix}, \quad \tilde{B}_p = \begin{bmatrix} B_- \\ B_+ D_{qp} \\ B_p \end{bmatrix}, \quad \tilde{C}_q = \begin{bmatrix} C_- & 0 & 0 \\ 0 & C_+ & 0 \\ 0 & 0 & C_q \end{bmatrix},$$

and

$$\tilde{B}_w = \begin{bmatrix} 0 \\ 0 \\ B_w \end{bmatrix}, \quad \text{and} \quad \tilde{C}_z = \begin{bmatrix} 0 & 0 & C_z \end{bmatrix}.$$

3 A numerical example

Consider a specific example of system (1), where

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \; B_p = \begin{bmatrix} 10 & 0 \\ 0 & 2/5 \end{bmatrix}, \; D_{qp} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix},$$

and C_q, B_w and C_z are identity matrices of appropriate sizes. The uncertainty Δ is a 2×2 diagonal, constant matrix, with diagonal entries in $[-\gamma, \gamma]$. For this system, the robust performance analysis problem considered is that of finding the largest $\mathbf{H}_2$ norm, over Δ, from w to z. Let us denote this quantity by η_{wc}.

Fig. 3 shows the plot of the various bounds on η_{wc} as a function of γ. The lower bound on η_{wc} obtained from Monte Carlo simulations is shown in solid lines. The dashed-dotted lines show the upper bound obtained with Lyapunov functions when no multipliers are used, while the upper bound with multipliers that add functional terms $e^{-t} * q(t)$ and $te^{-t} * q(t)$ to the Lyapunov function is shown in dotted lines. The dashed lines show the upper bound with Lyapunov functions with further functional terms $e^{-t} * p(t)$ and $te^{-t} * p(t)$.

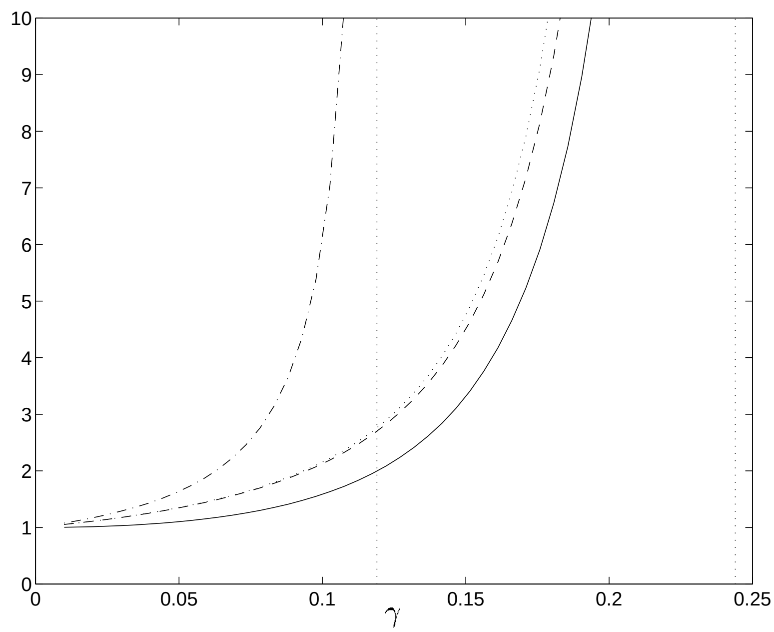

Figure 3: Plot of the bounds on the worst-case $\mathbf{H}_2$ norm as a function of γ.

Given the simplicity of this example, the Monte Carlo runs can be thought of yielding the exact value of η_{wc} with high likelihood. Then, it is evident from the plots that the upper bound on η_{wc}, obtained without any multipliers diverges from the actual value of η_{wc} rather rapidly as γ increases. Around $\gamma = 0.12$, this upper bound tends to infinity; the reason is that even robust stability cannot be established without any multipliers for $\gamma > 0.12$. The upper bound from Lyapunov functions for the augmented systems however, appear to be fairly tight in this example. As expected, the addition of additional dynamics leads to an improvement in the upper bound.

4 Conclusion

The techniques presented here provide a framework for generating Lyapunov functions for systems affected by structured nonlinearities and perturbations. The ideas presented extend small-gain, passivity, Popov, μ and real-μ stability tests to computing robust performance bounds. The underlying numerical methods do no require frequency sampling;

instead, the methods are based on state-space techniques and convex LMI optimization, and yield guaranteed bounds. However, the form of the Lyapunov function has to decided on, based on the information known about the uncertainty. Also, the question of how conservative the bounds obtained are, remains. The general framework presented here offers the potential for control synthesis to optimize robust performance bounds, using a "gain-scheduled" framework [24, 11, 2, 1].

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